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EXAMPLES FOR PRACTICE IN THE USE OF
SEVEN-FIGURE LOGARITHMS.



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EXAMPLES FOR PRACTICE

IN THE USE OF

SEVEN-FIGURE

LOGARITHMS

1✓

For the Use of Schools and Colleges

BY

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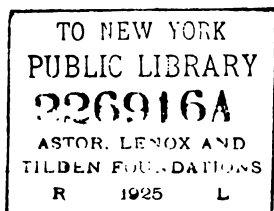
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PREFATORY NOTE.

CONSIDERABLE practice is required by most students before they acquire the accuracy which is absolutely necessary in the use of logarithms. According to my experience, mistakes are more frequently made by taking out a wrong logarithm than by blundering in the arithmetic. It would seem therefore that a student should always be expected to take out himself all the logarithms, whether of numbers or of Trigonometrical Functions, which are needed in the solution of any question; and that the practice, which has been so common, of supplying him with all these in addition to the data should be abandoned. It is mainly with the wish of bringing about this result that I have put together the accompanying collection of examples. As the answers are given in each case, this little book is the result of a very considerable amount of work. I have taken all pains to secure accuracy, but cannot hope to have completely succeeded, and shall be grateful to anyone who will point out to me any mistakes I may have made.

I have given what may be thought a disproportionate number of examples of the calculation of the parts of a tetrahedron when

the lengths of its edges are given. This was done, chiefly because I am convinced that no other kind of examples so soon teaches the habitual accuracy which ought to be attained, but also to some extent to give examples for the use of those who have occasion for practice in the solution of Spherical Triangles. In the calculation of each tetrahedron is involved the obtaining all the parts of four Spherical Triangles, and this has induced me, after considerable hesitation, to abstain from giving explicit examples for such solutions.

I have principally, but not exclusively, used Chambers' Mathematical Tables in my working out of these examples. Of course the seventh significant place in numbers, and the second decimal place in seconds of angles, are not absolutely to be relied upon. If this book should be used with tables of six or five-figure logarithms, the corresponding number of places should be struck off my results, the usual correction being made when necessary in the remaining final figure.

COOPER'S HILL COLLEGE, *Sept. 4th*, 1888.

EXAMPLES FOR PRACTICE IN THE USE OF SEVEN-FIGURE LOGARITHMS.

I.—Calculation of $L \sin$, $L \tan$, etc., when the angle is given, and conversely.

If it is required to find the $L \sin$, $L \cos$, $L \tan$, ..., of an angle not exactly given in the Tables, it is best to take out from the Tables the $L \sin$, $L \cos$, or $L \tan$ of that angle in the Tables which is next below the proposed angle; and then to calculate the "proportional part" for the seconds by a small sum in Practice worked at the side: this proportional part being then added (with the proper sign) to the logarithm taken out. Thus to calculate $L \tan 28^\circ 40' 49.2''$, $L \sin 27^\circ 45' 43.54''$, $L \cos 54^\circ 57' 58.2''$.

$\begin{array}{r} L \tan 28^\circ 40' = 9.7377714 \\ \text{diff. for } 49.2'' = \quad 2461 \\ \hline L \tan 28^\circ 40' 49.2'' = 9.7380175 \end{array}$	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $60'' = 3001$ </td> <td style="padding: 0 5px;"> 8 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $48'' = \frac{8}{10} = 2400$ </td> <td style="padding: 0 5px;"> 60 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $1.2'' = \frac{2}{100} =$ </td> <td style="padding: 0 5px;"> 2461 </td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-top: 1px solid black;"></td> </tr> </table>	$60'' = 3001$	8	$48'' = \frac{8}{10} = 2400$	60	$1.2'' = \frac{2}{100} =$	2461		
$60'' = 3001$	8								
$48'' = \frac{8}{10} = 2400$	60								
$1.2'' = \frac{2}{100} =$	2461								

$\begin{array}{r} L \sin 27^\circ 45' = 9.6680265 \\ \text{diff. for } 43.54'' = \quad 1742 \\ \hline L \sin 27^\circ 45' 23.54'' = 9.6682007 \end{array}$	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $60'' = 2400$ </td> <td style="padding: 0 5px;"> 6 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $42'' = \frac{7}{10} = 1680$ </td> <td style="padding: 0 5px;"> 1 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $1.5'' = \frac{1}{10} = 60$ </td> <td style="padding: 0 5px;"> 1742 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $.04'' = \frac{1}{2500} =$ </td> <td style="padding: 0 5px;"> 1 </td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-top: 1px solid black;"></td> </tr> </table>	$60'' = 2400$	6	$42'' = \frac{7}{10} = 1680$	1	$1.5'' = \frac{1}{10} = 60$	1742	$.04'' = \frac{1}{2500} =$	1		
$60'' = 2400$	6										
$42'' = \frac{7}{10} = 1680$	1										
$1.5'' = \frac{1}{10} = 60$	1742										
$.04'' = \frac{1}{2500} =$	1										

$\begin{array}{r} L \cos 54^\circ 57' = 9.7591321 \\ \text{diff. for } 58.2'' = \quad -1746 \\ \hline L \cos 54^\circ 57' 58.2'' = 9.7589575 \end{array}$	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $60'' = 1800$ </td> <td style="padding: 0 5px;"> 54 </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"> $1.8'' = \frac{3}{100} =$ </td> <td style="padding: 0 5px;"> 1746 </td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-top: 1px solid black;"></td> </tr> </table>	$60'' = 1800$	54	$1.8'' = \frac{3}{100} =$	1746		
$60'' = 1800$	54						
$1.8'' = \frac{3}{100} =$	1746						

Each of these examples might also be done by multiplying in each case the number of seconds by the quotient obtained by dividing by 60 the difference for 60". Thus, in the last example, this quotient is 30 and $58.2 \times 30 = 1746$. When the difference for 60" contains only three figures, this method is generally the shorter one. When the $L \sin$, $L \cos$, $L \tan$, ..., of an angle is given, and the angle is required, look out in the Tables the $L \sin$ (or $L \cos$ or $L \tan$, ...) nearest to the given one, whether greater or less, and write this under the given one, omitting those figures which are the same in both. Then take out the corresponding angle (putting + or - after it according as the number of seconds due to the difference is to be added or subtracted), and

the difference for 60"; and calculate the number of seconds by the usual proportion. Thus having given—

$$\begin{array}{l}
 (1) \quad \begin{array}{r} L \sin x = 9.8775429 \\ 48^\circ 58' - \} \\ 1100 \end{array} \quad \begin{array}{r} 1100 \\ 601 \\ \hline 172 \end{array} \quad \begin{array}{r} 10320(9.38'', \text{ or } x=48^\circ 57' 50.62''; \\ 42 \\ 90 \end{array} \\
 (2) \quad \begin{array}{r} L \cos x = 9.7076837 \\ 59^\circ 20' - \} \\ 2130 \end{array} \quad \begin{array}{r} 2130 \\ 064 \\ \hline 773 \end{array} \quad \begin{array}{r} 46380(21.78, \text{ or } x=59^\circ 19' 38.22''; \\ 378 \\ 165 \\ 159 \end{array} \\
 (3) \quad \begin{array}{r} L \sec x = 10.1324081 \\ 42^\circ 30' + \} \\ 1158 \end{array} \quad \begin{array}{r} 1158 \\ 391 \\ \hline 390 \end{array} \quad \begin{array}{r} 25400(18.48, \text{ or } x=42^\circ 30' 18.48''; \\ 9820 \\ 556 \\ 928 \end{array}
 \end{array}$$

The method of "proportional parts" must not be employed for the $L \sin$ or $L \tan$ of an angle less than $3^\circ 30'$, the methods for "small" angles being then employed; as also for the $L \cos$ or $L \sec$ of an angle greater than $86^\circ 30'$.

[Not many examples for working are given under this heading, as every example in all the subsequent sections will furnish one or more.]

EXAMPLES. I.

1. Find the $L \sin$ of the following angles :—

$$68^\circ 47' 41.5'', \quad 73^\circ 21' 23.91'', \quad 78^\circ 29' 43.85'', \quad 139^\circ 21' 10.74''.$$

[Ans. 9.9695517, 9.9814136, 9.9911858, 9.8138458.]

2. Find the $L \cos$ of the following angles :—

$$25^\circ 45' 33.61'', \quad 22^\circ 24' 19.04'', \quad 18^\circ 49' 46.32'', \quad 66^\circ 59' 38.97''.$$

[Ans. 9.9545452, 9.965912, 9.9761128, 9.59919823.]

3. Find the $L \tan$ of the following angles :—

$$23^\circ 48' 41.12'', \quad 46^\circ 0' 43.79'', \quad 66^\circ 11' 18.88'', \quad 43^\circ 59' 16.21''.$$

[Ans. 9.6447248, 10.0153473, 10.3552752, 9.9846527.]

4. Find the value of x from each of the following equations :—

$$(\alpha) \quad L \sin x = 9.5740674, \quad L \sin x = 9.8091259, \quad L \sin x = 9.8281920, \quad L \sin x = 9.9860122;$$

$$(\beta) \quad L \tan x = 9.3600738, \quad L \tan x = 9.8353762, \quad L \tan x = 10.0562416, \quad L \tan x = 10.1321843;$$

$$(\gamma) \quad L \cos x = 9.9676225, \quad L \cos x = 9.8610172, \quad L \cos x = 9.7544211, \quad L \cos x = 9.3701332.$$

$$\begin{array}{l}
 \text{[Ans. } \left\{ \begin{array}{l} (\alpha) \quad 22^\circ 1' 34.47'', \quad 40^\circ 7' 2.69'', \quad 42^\circ 19' 13.01'', \quad 75^\circ 32' 9.83''; \\ (\beta) \quad 12^\circ 54' 18.65'', \quad 34^\circ 23' 30.51'', \quad 48^\circ 41' 58.59'', \quad 53^\circ 35' 16.23''; \\ (\gamma) \quad 21^\circ 51' 1.23'', \quad 43^\circ 26' 12.02'', \quad 59^\circ 22' 56.98'', \quad 76^\circ 26' 17.35''. \end{array} \right. \\
 \end{array}$$

5. Find the values of the following fractions :—

$$(1) \quad \frac{\sin 58^\circ 9' 20.9'' \sin 49^\circ 26' 47.92'' \sin 65^\circ 18' 24.84''}{\sin 57^\circ 11' 25.7'' \sin 53^\circ 17' 18.06'' \sin 60^\circ 29' 59.54''}$$

$$(2) \quad \frac{\sin 57^\circ 11' 25.7'' \sin 61^\circ 20' 39.56'' \sin 68^\circ 33' 21.04''}{\sin 58^\circ 9' 20.9'' \sin 57^\circ 30' 9.46'' \sin 73^\circ 21' 46.38''}$$

$$(3) \quad \frac{\sin 61^\circ 24' 17.1'' \sin 57^\circ 30' 9.46'' \sin 60^\circ 29' 59.54''}{\sin 53^\circ 56' 29.5'' \sin 61^\circ 20' 39.56'' \sin 65^\circ 18' 24.84''}$$

$$(4) \quad \frac{\sin 53^\circ 56' 29.5'' \sin 53^\circ 17' 18.06'' \sin 73^\circ 21' 46.38''}{\sin 61^\circ 24' 17.1'' \sin 49^\circ 26' 47.92'' \sin 68^\circ 33' 21.04''}$$

[Each fraction=1.]

6. Prove the truth of the equations :—

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = \frac{2\sqrt{\sin s \sin(s-a) \sin(s-b) \sin(s-c)}}{\sin a \sin b \sin c}, \quad (2s \equiv a + b + c),$$

for the following systems :—

$$\begin{aligned} (1) \quad & \left\{ \begin{array}{l} a = 27^\circ 28' 41.02'', \quad A = 49^\circ 24' 1.48'' \\ b = 33^\circ 18' 34.24'', \quad B = 64^\circ 38' 45.10'' \\ c = 35^\circ 42' 29.32'', \quad C = 73^\circ 49' 49.80'' \end{array} \right\}; \quad (2) \quad \left\{ \begin{array}{l} a = 50^\circ 0' 30.76'', \quad A = 49^\circ 24' 1.48'' \\ b = 62^\circ 59' 51.52'', \quad B = 62^\circ 0' 25.14'' \\ c = 81^\circ 9' 58.50'', \quad C = 101^\circ 40' 57.08'' \end{array} \right\}; \\ (3) \quad & \left\{ \begin{array}{l} a = 81^\circ 17' 39.16'', \quad A = 77^\circ 15' 8.76'' \\ b = 66^\circ 19' 31.38'', \quad B = 64^\circ 38' 45.10'' \\ c = 97^\circ 2' 17.90'', \quad C = 101^\circ 40' 57.08'' \end{array} \right\}; \quad (4) \quad \left\{ \begin{array}{l} a = 65^\circ 31' 27.26'', \quad A = 77^\circ 15' 8.76'' \\ b = 55^\circ 29' 1.08'', \quad B = 62^\circ 0' 25.14'' \\ c = 63^\circ 39' 57.86'', \quad C = 73^\circ 49' 49.80'' \end{array} \right\}; \end{aligned}$$

also prove that the value of each member of the equations is (1) 1.64556, (2) .991042, (3) .9867222, (4) 1.071654.

II.—Calculation of $\sqrt{a^2 + b^2}$, when a, b are given numbers.

Taking u to denote $\sqrt{a^2 + b^2}$, then if θ be an angle determined by the equation $\tan \theta = \frac{b}{a}$, $u = a \sec \theta = b \operatorname{cosec} \theta$. The value of u may be determined from either of these equations; if both be used, the calculation is thereby checked. The work may be conveniently arranged as in the following example.

$a = 47993.27$ $b = 38932.89$	$\log b = 4.5903167$ $\log a = 4.6811804$ $L \tan \theta = 9.9091363$ $39^\circ 3' - \}$ $2582 \}$ 77	$4620(1.79$ 2038 2306 $\theta = 39^\circ 2' 58.21''$	$\log \sec \theta = .1098046$ -31 $\log a = 4.6811804$ $\log u = 4.7909819$	$1025 = 60''$ $1.8 = 1\frac{3}{10} = 30$ $-.01 = -$ 31	8 2
$\log \operatorname{cosec} \theta = .2006606$ $+46$ $\log b = 4.5903167$ $\log u = 4.7909819$		$1558 = 60''$ $1.8 = 46$ $-.01 = -$ 46			7 3

The two calculations agree in giving $\log u = 4.7909819$, which is therefore almost certainly correct. This gives $u = 61799.07$.

EXAMPLES. II.

DATA.	ANSWERS.
1. $a = 571.4284, \quad b = 428.5713.$	714.2854 ($\theta = 36^\circ 52' 11.65''$).
2. $a = 93921.58, \quad b = 37917.43.$	101286.7 ($\theta = 21^\circ 59' 4.87''$).
3. $a = 4285.714, \quad b = 5714.285.$	7142.855 ($\theta = 36^\circ 52' 11.65''$).
4. $a = 23727.92, \quad b = 7873.11.$	25000.01 ($\theta = 18^\circ 21' 22.5''$).
5. $a = 7139.247, \quad b = 3143.892.$	7800.827 ($\theta = 23^\circ 46' 1.68''$).
6. $a = 52791.39, \quad b = 37387.41.$	64689.64 ($\theta = 35^\circ 18' 23.46''$).
7. $a = 171428.4, \quad b = 169999.8.$	241428.3 ($\theta = 44^\circ 45' 36.92''$).
8. $a = 3081.708, \quad b = 212.7818.$	3089.046 ($\theta = 3^\circ 56' 59.34''$).
9. $a = 9853.08, \quad b = 2127.818.$	10080.22 ($\theta = 12^\circ 11' 10.14''$).
10. $a = 13790.4, \quad b = 13790.3.$	19502.5 ($\theta = 44^\circ 59' 59.26''$).
11. $a = 803.761, \quad b = 803.76.$	1136.689 ($\theta = 44^\circ 59' 59.88''$).

EXAMPLES. II.—Continued.

DATA.	ANSWERS.
12. $a = 7118.12$, $b = 606.13$.	7143.878 ($\theta = 4^\circ 52' 1.82''$).
13. $a = 34285.68$, $b = 9999.99$.	35714.25 ($\theta = 16^\circ 15' 36.75''$).
14. $a = 7372.46$, $b = 6989.37$.	10158.95 ($\theta = 43^\circ 28' 19.35''$).
15. $a = 1185.184$, $b = 345.6796$.	1234.567 ($\theta = 16^\circ 15' 36.88''$).
16. $a = 273.0004$, $b = 135.9992$.	305. ($\theta = 26^\circ 28' 51.17''$).
17. $a = 272.9$, $b = 136.2022$.	305.0008 ($\theta = 26^\circ 31' 24.3''$).
18. $a = 5488.42$, $b = 2273.378$.	5940.623 ($\theta = 22^\circ 30'$).
19. $a = 378.4125$, $b = 487.4775$.	615.37 ($\theta = 52^\circ 5' 4.83''$).
20. $a = 27891.83$, $b = 36789.17$.	46167.06 ($\theta = 52^\circ 49' 56.27''$).
21. $a = 262.087$, $b = 151.316$.	302.632 ($\theta = 29^\circ 59' 59.98''$).
22. $a = 56471.9$, $b = 97812.2$.	112943.8 ($\theta = 60^\circ$).
23. $a = 8055.837$, $b = 5924.819$.	10000 ($\theta = 36^\circ 20'$).
24. $a = 7601.54$, $b = 6497.43$.	10000 ($\theta = 40^\circ 31' 20''$).
25. $a = 7968.81$, $b = 23695.95$.	25000 ($\theta = 71^\circ 24' 45.02''$).
26. $a = 1236.113$, $b = 716.1294$.	1428.572 ($\theta = 30^\circ 5' 7.5''$).
27. $a = 74511.63$, $b = 19109.62$.	76923.095 ($\theta = 14^\circ 23' 3.6''$).
28. $a = 4313.546$, $b = 3999.424$.	5882.351 ($\theta = 42^\circ 50' 9.6''$).
29. $a = 4127.817$, $b = 1365.548$.	4347.825 ($\theta = 18^\circ 18' 18.17''$).
30. $a = 437.7493$, $b = 796.7571$.	909.0909 ($\theta = 61^\circ 12' 54''$).
31. $a = 6151.67$, $b = 1104.292$.	6250.001 ($\theta = 10^\circ 10' 36.6''$).
32. $a = 4761.903$, $b = 13468.55$.	14285.67 ($\theta = 70^\circ 31' 42.88''$).

III.—Solution of Equations of the form $a \cos x + b \sin x = c$.

The two values of x which satisfy the equation $a \cos x + b \sin x = c$ being denoted by α , β , $\frac{1}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{1}{b} \sin \frac{1}{2}(\alpha + \beta) = \frac{1}{c} \cos \frac{1}{2}(\alpha - \beta)$. First find $\frac{1}{2}(\alpha + \beta)$ from the equation $\tan \frac{1}{2}(\alpha + \beta) = \frac{b}{a}$ and then $\frac{1}{2}(\alpha - \beta)$ from either or both of the equations $\cos \frac{1}{2}(\alpha - \beta) = \frac{c}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{c}{b} \sin \frac{1}{2}(\alpha + \beta)$. Writing the equation so that c is positive, then if a be negative and b positive, put $x = \pi - y$; if a be positive and b negative, put $x = -y$; and if both a and b be negative, put $x = \pi + y$. We shall thus always have an equation in which the coefficients are all positive. Arrange the work according to the following pattern.

$a=27839$ $b=29273$ $c=28565$	$14.4664672 \dots (10 + \log b)$ $-4.4446536 \dots (\log a)$ $L \tan \frac{\alpha + \beta}{2} = 10.0218136$ $46^\circ 26' + \left. \begin{array}{l} 2 \\ 253\phi \end{array} \right\} \frac{7380}{756 \times 6\phi}$ $4536(17.93)$ 2006 235 73	$9.8383441 \left. \begin{array}{l} \\ -397 \end{array} \right\}$ 4.4558342 -4.4446536 9.8494850 $= L \cos \frac{1}{2}(\alpha - \beta)$ $= L \cos 45^\circ$	(1) $1329=60''$ $18=398$ $-.06=-1$ $-.01=-$ 397 7 3 2
		$9.8600821 \left. \begin{array}{l} \\ 359 \end{array} \right\}$ 4.4558342 -4.4664672 9.8494850 $= L \cos \frac{1}{2}(\alpha - \beta)$	(2) $1201=60''=20\frac{1}{8} \times 60$ 358.6 3 359

Thus
$$\left. \begin{aligned} \frac{a+\beta}{2} &= 46^{\circ}26'17.93'', \\ \frac{a-\beta}{2} &= 45^{\circ}, \end{aligned} \right\} \text{whence } a = 91^{\circ}26'17.93'', \beta = 1^{\circ}26'17.93''.$$

If the values of a, b, c be such that $c^2 > a^2 + b^2$, the values of x will be impossible. Should such data be given, the value found for $L \cos \frac{1}{2}(a - \beta)$ would be greater than 10, and the corresponding angle impossible.

EXAMPLES. III.

DATA.	ANSWERS.
1. $a = 11175.3, b = 5600.24, c = 10548.25.$	$\alpha = 59^{\circ}4', \beta = 5^{\circ}50'.$
2. $a = 58533.32, b = 49909.95, c = 62137.14.$	$\alpha = 76^{\circ}34'24.7'', \beta = 4^{\circ}20'.$
3. $a = 67313.97, b = 58129.13, c = 62721.45.$	$\alpha = 85^{\circ}57'38.3'', \beta = -4^{\circ}20'9.6''.$
4. $a = 39713.76, b = 29254.24, c = 18397.98.$	$\alpha = 104^{\circ}28'34.01'', \beta = -31^{\circ}43'24.07''.$
5. $a = 6342.667, b = 3914.217, c = 6549.328.$	$\alpha = 60^{\circ}11'27.25'', \beta = 3^{\circ}10'7.07''.$
6. $a = 549.1238, b = 508.1741, c = 618.3272.$	$\alpha = 77^{\circ}2'50.77'', \beta = 8^{\circ}30'59.71''.$
7. $a = 35.67812, b = 28.73407, c = 40.13598.$	$\alpha = 67^{\circ}40'1.8'', \beta = 10^{\circ}1'36''.$
8. $a = 7891.293, b = 889.312, c = 7926.027.$	$\alpha = 9^{\circ}58'39.75'', \beta = 2^{\circ}52'55.13''.$
9. $a = 3712.484, b = 1139.115, c = 3795.726.$	$\alpha = 29^{\circ}14'59.21'', \beta = 4^{\circ}51'56.79''.$
10. $a = 861.3982, b = 116.4593, c = 869.154.$	$\alpha = 8^{\circ}28'58.52'', \beta = 6^{\circ}54'58.52''.$
11. $a = 25136.85, b = 23846.16, c = 27389.27.$	$\alpha = 81^{\circ}15'31.62'', \beta = 6^{\circ}23'20.84''.$
12. $a = 1790.009, b = 2811.935, c = 2956.161.$	$\alpha = 85^{\circ}2'25.2'', \beta = 30''.$
13. $a = 78913.47, b = 59134.72, c = 67314.95.$	$\alpha = 83^{\circ}47'49.92'', \beta = -10^{\circ}6'15.32''.$
14. $a = 3581.72, b = 3312.93, c = 4645.07.$	$\alpha = 60^{\circ}34'48.14'', \beta = 24^{\circ}57'17.40''.$
15. $a = 428.5714, b = 571.4286, c = 714.2857.$	$\alpha = 53^{\circ}7'48.35'', \beta = 53^{\circ}7'48.35''.$
16. $a = 20785.2, b = 20611.99, c = 20958.41.$	$\alpha = 89^{\circ}2'13.18'', \beta = 0^{\circ}29'0.76''.$
17. $a = 9999.79, b = 8654.33, c = 11235.84.$	$\alpha = 72^{\circ}42'19.56'', \beta = 9^{\circ}2'37.46''.$
18. $a = 37912.37, b = 28776.52, c = 39148.21.$	$\alpha = 71^{\circ}51'49.02'', \beta = 2^{\circ}32'7.22''.$
19. $a = 84513.27, b = 9837.12, c = 79125.64.$	$\alpha = 28^{\circ}12'10.98'', \beta = -14^{\circ}55'28.70''.$
20. $a = 831.2798, b = 415.6399, c = 888.8931.$	$\alpha = 44^{\circ}25'31.66'', \beta = 8^{\circ}22'32.54''.$
21. $a = 813908.5, b = 600305.1, c = 1000000.$	$\alpha = 45^{\circ}0'0.04'', \beta = 27^{\circ}49'18.4''.$
22. $a = 7933.533, b = 6087.614, c = 9914.449.$	$\alpha = 45'', \beta = 30''.$
23. $a = 7261.248, b = 6875.629, c = 9915.867.$	$\alpha = 50^{\circ}52'30'', \beta = 36^{\circ}0'0.0''.$
24. $a = 35170.77, b = 35538.95, c = 48363.02.$	$\alpha = 60'', \beta = 30^{\circ}35'48.0''.$
25. $a = 786.1347, b = 563.9187, c = 803.4125.$	$\alpha = 69^{\circ}31'40.06'', \beta = 1^{\circ}46'41.84''.$
26. $a = 43759.16, b = 24189.58, c = 44576.71.$	$\alpha = 92^{\circ}0'0.00'', \beta = -34^{\circ}8'0.00''.$
27. $a = 137904, b = 137903, c = 195025.$	$\alpha = 44^{\circ}59'59.24'', \beta = 44^{\circ}59'59.24''.$
28. $a = 803.76, b = 803.761, c = 1136.689.$	$\alpha = 45^{\circ}0'0.12'', \beta = 45^{\circ}0'0.12''.$
29. $a = 70, b = 24, c = 37.$	$\alpha = 78^{\circ}55'28.72'', \beta = -41^{\circ}4'31.28''.$
30. $a = 70, b = 24, c = 64.08587.$	$\alpha = 48^{\circ}55'28.72'', \beta = -11^{\circ}4'31.28''.$
31. $a = 39405.98, b = 17622.89, c = 21583.55.$	$\alpha = 215^{\circ}54'18.56'', \beta = 95^{\circ}54'18.6''.$
32. $a = 97.8122, b = 56.4719, c = 112.9438.$	$\alpha = 30'', \beta = 30''.$
33. $a = 27442, b = 11366.89, c = 29703.12.$	$\alpha = 22^{\circ}30', \beta = 22^{\circ}30'.$
34. $a = 5488.43, b = 2273.382, c = 5940.635.$	$\alpha = 22^{\circ}30', \beta = 22^{\circ}30'.$
35. $a = 43155.16, b = 68971.5, c = 71859.22.$	$\alpha = 85^{\circ}55'55'', \beta = 30''.$
36. $a = 395.76, b = 228.48, c = 323.12.$	$\alpha = 75^{\circ}0'3.18'', \beta = -15^{\circ}0'12.68''.$
37. $a = 193.19, b = 198.79, c = 196.01.$	$\alpha = 90^{\circ}49'6.6'', \beta = 0^{\circ}49'6.6''.$
38. $a = 27.839, b = 28.273, c = 28.565.$	$\alpha = 89^{\circ}23'44.83'', \beta = 1^{\circ}29'25.83''.$
39. $a = 239.9, b = 3992.1, c = 3568.9.$	$\alpha = 113^{\circ}23'13.59'', \beta = 59^{\circ}44'5.71''.$
40. $a = 11.599, b = 49.121, c = 35.689.$	$\alpha = 121^{\circ}42'50.56'', \beta = 31^{\circ}42'50.56''.$
41. $a = 3496.7, b = 6399.7, c = 5125.7.$	$\alpha = 106^{\circ}41'30.68'', \beta = 16^{\circ}0'18.28''.$
42. $a = 34967, b = 63497, c = 51257.$	$\alpha = 106^{\circ}9'32.31'', \beta = 16^{\circ}9'32.31''.$

IV.—Calculation of $\sqrt{b^2 + c^2 - 2bc \cos A}$.

Denoting the expression by u , its value may be calculated by any one of the three following methods :—

- (1) $u_1 = (b+c) \cos \theta$, where $\sin \theta = \frac{2\sqrt{bc} \cos \frac{1}{2}A}{b+c}$;
 (2) $u_2 = (b-c) \sec \theta$, where $\tan \theta = \frac{2\sqrt{bc}}{b-c} \sin \frac{1}{2}A$; ($b > c$)
 (3) $u_3 = (b+c) \sin \frac{1}{2}A \sec \theta$, where $\tan \theta = \frac{b-c}{b+c} \cot \frac{1}{2}A$;

The calculation of one example by each of these three methods is subjoined.

DATA.			
$b=12369.12$	$\log b=4.0923388$		
$c=4123.04$	$\log c=3.6152175$		
$A=40^\circ 55' 55''$		$2)7.7075563$	
$b+c=16492.16$	$\log \sqrt{bc}=3.8537782$		
$b-c=8246.08$	$\log (b+c)=4.2172775$		
$\frac{A}{2}=20^\circ 27' 57.5''$	$\log (b-c)=3.9162475$		
$\frac{A}{2}=20^\circ 28' -2.5''$			
(1)			
$\begin{array}{r} .30103 \\ 3.8537782 \\ 9.9716820 \\ 19 \\ -4.2172775 \end{array}$	$\begin{array}{r} 471 \\ 2.4=18 \quad 8 \\ .1=7 \\ 2.5=19 \quad 5 \end{array}$	$\begin{array}{r} \log \cos \theta = \bar{1}.7667739 \\ 432 \\ \log (b+c)=4.2172775 \\ \log u_1=3.9840946 \\ 05 \\ 9640.3 \quad 41 \\ 9=41 \\ u_1=9640.390 \end{array}$	$\begin{array}{r} 1753 \\ 10=292 \quad 2 \\ 4.8=140 \quad 2 \\ -.03=1 \quad 2 \\ 14.84=432 \end{array}$
$L \sin \theta = 9.9092146$	$)1350(14.84''$		
$54^\circ 14' - \left. \begin{array}{r} 371 \\ 910 \end{array} \right\} 225$	$\begin{array}{r} 440 \\ 76 \\ 32 \end{array}$		
(2)			
$\begin{array}{r} .3010300 \\ 3.8537782 \\ 9.5430489 \\ -141 \\ -3.9162475 \end{array}$	$\begin{array}{r} 3386 \\ 2 = 113 \\ .5 = 28 \\ 2.5 = 141 \end{array}$	$\begin{array}{r} \log \sec \theta = .0678489 \\ -16 \\ \log (b-c)=3.9162475 \\ \log u_2=3.9840948 \\ u_2=9640.394 \end{array}$	$\begin{array}{r} 765 \\ 1.2=15 \quad 3 \\ .02= \quad 2 \\ 15 \quad 5 \end{array}$
$L \tan \theta = 9.7821955$	$)3480(1.22$		
$31^\circ 12' - \left. \begin{array}{r} 2013 \\ 2851 \end{array} \right\} 58$	$\begin{array}{r} 629 \\ 588 \end{array}$		
(3)			
$\begin{array}{r} 3.9162475 \\ 10.4280331 \\ 161 \\ -4.2172775 \end{array}$	$\begin{array}{r} 3858 \\ 2 = 128 \quad 6 \\ .5 = 32 \quad 2 \\ 2.5 = 161 \end{array}$	$\begin{array}{r} \log \sec \theta = .2232324 \\ -498 \\ \log \sin \frac{1}{2}A = \bar{1}.5436348 \\ \log (b+c)=4.2172775 \\ \log u_3=3.9840949 \\ u_3=9640.396 \end{array}$	$\begin{array}{r} 1693 \\ 15 = 423 \quad 3 \\ 2 = 56 \quad 4 \\ .66= 18 \quad 6 \\ 17.66=498 \end{array}$
$L \tan \theta = 10.1270192$	$)46560(17.66$		
$53^\circ 16' - \left. \begin{array}{r} 968 \\ 2636 \end{array} \right\} 776$	$\begin{array}{r} 2020 \\ 1748 \\ 1664 \end{array}$		

The three values of $\log u$, found according to the three methods, will generally have slight differences in the last place. In the examples given, the answers will exhibit the three values of u , denoted by u_1, u_2, u_3 ; and the mean of these is taken for the value of u . Thus, in the one calculated, $u_1 = 9640.390, u_2 = 9640.394, u_3 = 9640.396, u = 9640.393$.

EXAMPLES. IV.

DATA.	ANSWERS.
1. $b=45123.76, c=34179.24, A=24^\circ 57' 18.6''$.	$u_1=20193.17, u_2=u_3=20193.20, u=20193.18$.
2. $b=2357.126, c=2075.352, A=67^\circ 31' 55''$.	$u_1=2474.689, u_2=u_3=2474.691, u=2474.69$.
3. $b=2357.126, c=2075.382, A=67^\circ 31' 55''$.	$u_1=u_2=u_3=u=2474.705$.
4. $b=47392.16, c=25497.83, A=56^\circ 37' 55.2''$.	$u_1=39583.89, u_2=u_3=39583.92, u=39583.9$.
5. $b=38914.86, c=34819.73, A=120^\circ$.	$u_1=63888.85, u_2=63888.91, u_3=63888.89,$ $u=63888.88$.
6. $b=73869.13, c=9347.25, A=60^\circ$.	$u_1=69667.42, u_2=u_3=69667.40, u=69667.41$.
7. $b=543.2971, c=276.8139, A=36^\circ 55' 57.6''$.	$u_1=362.4475, u_2=362.4477, u_3=362.447,$ $u=362.4474$.
8. $b=47394.91, c=21382.75, A=26^\circ 39' 45''$.	$u_1=29869.01, u_2=u_3=29869.03, u=29869.02$.
9. $b=7984.637, c=4138.363, A=31^\circ 37' 52.8''$.	$u_1=4961.009, u_2=u_3=4961.012, u=4961.011$.
10. $b=73182.2, c=47297.4, A=39^\circ 19' 5.62''$.	$u_1=47297.42, u_2=u_3=47297.39, u=c=47297.4$.
11. $b=670.344, c=586.551, A=120^\circ$.	$u_1=u_2=u_3=u=1089.309$.
12. $b=3791.298, c=413.115, A=4^\circ 30'$.	$u_1=3379.611, u_2=3379.69, u_3=3379.612,$ $u=3379.611$.
13. $b=3789.371, c=1134.199, A=23^\circ 6' 16.8''$.	$u_1=2781.982, u_2=u_3=2781.98, u=2781.981$.
14. $b=890.8565, c=534.1539, A=53^\circ 7' 48.38''$.	$u_1=718.6850, u_2=718.6852(=u), u_3=718.6855$.

Other examples for practice may be taken from VII.

V.—Triangles in which are given two angles and one side.

Let a be the given length of one side; then whichever two of the three angles are given, the third is at once found from the equation $A + B + C = 180^\circ$; then find R the radius of the circumscribed circle from the equation $2R = a/\sin A$, and the other sides b, c from the equations $b = 2R \sin B, c = 2R \sin C$. The radii of the inscribed and escribed circles r, r_1, r_2, r_3 , if required, may best be found from the equations $r = (s - a) \tan \frac{A}{2}, r_1 = s \tan \frac{A}{2}, r_2 = (s - c) \cot \frac{A}{2}, r_3 = (s - b) \cot \frac{A}{2}$, or the corresponding system involving $\tan \frac{B}{2}$ or $\tan \frac{C}{2}$, taking that one of the three half angles which is nearest to 45° . The equation $r_1 + r_2 + r_3 - r = 4R$ should then be employed to check the calculations. An example is worked out at full length below.

DATA.		
$a=27837.93$	$\log a = 4.4446369$	277
$A=77^\circ 36' 57.6''$	$-\log \sin A = -1.9897766$	
$B=54^\circ 22' 12.48''$	-11	2.4= $1\frac{1}{10}$ =11 1
hence $C=48^\circ 0' 49.92''$	$\log 2R = 4.4548614$	
	01	28501.09.
	13	

$\log 2R=4.4548614$ $\log \sin B=1.9099634$ $\log b=4.3648436$ $\underline{23165.60.}$ 113	905 $12''=181$ $.48=7\ 2$	$\log 2R=4.4548614$ $\log \sin C=1.8710735$ $\log c=4.3260295$ $\underline{21185.05.}$ 10	1137 $48=909\ 6$ $1.8=34\ 1$ $12=2\ 3$
$a=27837.93$ $b=23165.60$ $c=21185.05$ $2s=72188.58$ $s=36094.29$	$s-a=8256.36$ $s-b=12928.69$ $s-c=14909.24$ $s=36094.29$	$\frac{A}{2}=38^\circ 48' 28.8''$ $L \tan \frac{1}{2}A=9.9052672$ $\log \tan \frac{1}{2}A=\bar{1}.9053914$ $\log \cot \frac{1}{2}A=0.0946086$	$\log s-a=3.9167886$ $\log s-b=4.1115545$ $\log s-c=4.1734555$ $\log s=4.5574385$
$\log s-a=3.9167886$ $\log \tan \frac{1}{2}A=\bar{1}.9053914$ $\log r=3.8221800$ $\underline{6640.182.}$ -12	$\log s=4.5574385$ $\log \tan \frac{1}{2}A=\bar{1}.9053914$ $\log r_1=4.4628299$ $\underline{29028.85.}$ 128	$\log s-c=4.1734555$ $\log \cot \frac{1}{2}A=0.0946086$ $\log r_2=4.2680641$ $\underline{18538.05.}$ 12	$\log s-b=4.1115545$ $\log \cot \frac{1}{2}A=0.0946086$ $\log r_3=4.2061631$ $\underline{16075.45.}$ 121
ANSWERS. $C=48^\circ 0' 49.92''$ $b=23165.66$ $c=21185.05$ $R=14250.54$		CHECK. $-r=-6640.18$ $r_1=29028.85$ $r_2=18538.05$ $r_3=16075.45$ $4R=57002.17$ $R=14250.54$	
$r=6640.18$ $r_1=29028.85$ $r_2=18538.05$ $r_3=16075.45$			

The distance between the circumcentre, orthocentre, and incentre are to be found, when required, from the equations $\Delta^2 = R^2 - 2Rr$, $\Delta_2^2 = R^2 + 2\rho^2$, $\Delta_3^2 = \rho^2 + 2r^2$, where $\rho^2 = -4R^2 \cos A \cos B \cos C$.

EXAMPLES. V.

DATA.	ANSWERS.
1. $a=7$ $B=38^\circ 36'$ $C=129^\circ 23'$	$A=12^\circ 1'$ $b=20.97616$ $c=25.98715$ $R=16.81107$ $r=2.103095$ $r_1=2.839854$ $r_2=9.448844$ $r_3=57.05867$
2. $a=86602.54$ $B=80^\circ 42' 17.3''$ $C=39^\circ 17' 42.7''$	$A=60^\circ$ $b=98686.93$ $c=63331.6$ $R=50000$ $r=21770.725$ $r_1=71771.02$ $r_2=105618.62$ $r_3=44381.38$
3. $a=91035.91$ $B=37^\circ 43'$ $C=66^\circ 43' 41.9''$	$A=65^\circ 33' 18.1''$ $b=61175.72$ $c=91864.15$ $R=50000$ $r=20788.34$ $r_1=88339.88$ $r_2=41683.78$ $r_3=90764.7$

EXAMPLES. V.—Continued.

DATA.

ANSWERS.

4. $a=96840.39$ $B=37^{\circ}43'$ $C=66^{\circ}43'28.3''$	$A=75^{\circ}33'31.7''$ $b=61175.71$ $c=91861.56$ $R=50000$	$r=21779.2$ $r_1=96840.57$ $r_2=42674.64$ $r_3=82264$
5. $a=11935.26$ $B=41^{\circ}24'34.3''$ $C=24^{\circ}17'42.3''$	$A=114^{\circ}17'43.4''$ $b=8661.392$ $c=5387.97$ $R=6547.5$	$r=1636.927$ $r_1=20119.62$ $r_2=4910.528$ $r_3=2796.707$
6. $a=22733.78$ $B=90^{\circ}$ $C=67^{\circ}30'$	$A=22^{\circ}30'$ $b=59406.23$ $c=54884.19$ $R=29703.115$	$r=9105.87$ $r_1=13627.91$ $r_2=68512.1$ $r_3=45778.32$
7. $a=6347.287$ $A=78^{\circ}33'14.7''$ $B=39^{\circ}59'43.5''$	$C=61^{\circ}27'1.8''$ $b=4162.347$ $c=6404.116$ $R=3238.04$	$r=1725.267$ $r_1=6916.216$ $r_2=2510.033$ $r_3=5251.173$
8. $a=12791.38$ $A=62^{\circ}37'15.5''$ $B=68^{\circ}25'37.24''$	$C=48^{\circ}57'7.26''$ $b=13395.9$ $c=10863.65$ $R=7202.49$	$r=3487.813$ $r_1=11268.31$ $r_2=12596.28$ $r_3=8433.18$
9. $a=7674.893$ $A=97^{\circ}18'47.4''$ $C=40^{\circ}14'33.5''$	$B=42^{\circ}26'39.1''$ $b=5222.048$ $c=4998.842$ $R=3868.919$	$r=1446.811$ $r_1=10169.62$ $r_2=3474.63$ $r_3=3278.239$
10. $a=44569.33$ $A=80^{\circ}32'57.54''$ $C=45^{\circ}36'2.36''$	$B=53^{\circ}51'0.1$ $b=36483.84$ $c=32282.09$ $R=22591.29$	$r=10250.9$ $r_1=48014.57$ $r_2=28780.24$ $r_3=23821.26$
11. $a=8984.798$ $A=72^{\circ}48'48.6''$ $C=36^{\circ}36'12.72''$	$B=70^{\circ}34'58.68''$ $b=8869.832$ $c=5607.804$ $R=4702.37$	$r=2025.334$ $r_1=8651.132$ $r_2=8303.546$ $r_3=3880.131$
12. $a=381.437$ $A=67^{\circ}10'25.8''$ $C=75^{\circ}31'14.5''$	$B=37^{\circ}18'19.7''$ $b=250.818$ $c=400.7024$ $R=206.9235$	$r=89.677$ $r_1=342.9778$ $r_2=174.3436$ $r_3=400.0496$
13. $a=176.01$ $A=66^{\circ}19'31.38''$ $B=63^{\circ}39'57.86''$	$C=58^{\circ}0'30.76''$ $b=172.24$ $c=147.24$ $R=96.0921$	$r=46.8754$ $r_1=161.8895$ $r_2=153.8063$ $r_3=115.5479$
14. $a=443.387$ $B=27^{\circ}28'41.02''$ $C=55^{\circ}29'1.08''$	$A=97^{\circ}2'17.90''$ $b=206.136$ $c=368.109$ $R=223.3767$	$r=74.00344$ $r_1=575.4995$ $r_2=124.4027$ $r_3=267.6083$
15. $a=1266.82$ $A=81^{\circ}9'58.5''$ $C=33^{\circ}18'34.24''$	$B=65^{\circ}31'27.26''$ $b=1166.82$ $c=704.04$ $R=641.0137$	$r=258.7082$ $r_1=1343.8572$ $r_2=1009.5812$ $r_3=469.3244$

EXAMPLES. V.—Continued.

DATA.

ANSWERS.

16. $a=525.069$ $A=81^{\circ}17'39.16''$ $B=62^{\circ}59'59.52''$	$C=35^{\circ}42'29.32''$ $b=473.283$ $c=310.032$ $R=265.5946$	$r=110.8561$ $r_1=561.644$ $r_2=400.8712$ $r_3=210.7192$
17. $a=122.073$ $A=64^{\circ}31'45.28''$ $B=65^{\circ}48'27.70''$	$C=49^{\circ}39'47.02''$ $b=123.34$ $c=103.068$ $R=67.60763$	$r=32.93385$ $r_1=109.9998$ $r_2=112.73784$ $r_3=80.62681$
18. $a=786.33$ $B=21^{\circ}33'9.62''$ $C=57^{\circ}14'20.48''$	$A=101^{\circ}12'29.90''$ $b=294.48$ $c=674.11$ $R=400.8097$	$r=110.9599$ $r_1=1068.3954$ $r_2=167.0089$ $r_3=478.7944$
19. $a=2358.99$ $A=86^{\circ}10'22.64''$ $B=67^{\circ}33'33.80''$	$C=26^{\circ}16'3.56''$ $b=2155.23$ $c=1040.34$ $R=1182.131$	$r=408.0798$ $r_1=2614.54$ $r_2=1068.843$ $r_3=652.2222$
20. $a=70.48$ $A=28^{\circ}46'16.82''$ $B=67^{\circ}1'50.08''$	$C=84^{\circ}11'53.10''$ $b=134.822$ $c=145.682$ $R=73.21606$	$r=26.9545$ $r_1=45.0119$ $r_2=110.223$ $r_3=158.5638$
21. $a=4322.56$ $A=72^{\circ}19'11.54''$ $C=51^{\circ}21'10.96''$	$B=56^{\circ}19'37.50''$ $b=3775.64$ $c=3543.32$ $R=2268.424$	$r=1094.909$ $r_1=4253.906$ $r_2=3116.297$ $r_3=2798.406$
22. $a=1204.66$ $B=74^{\circ}14'44.72''$ $C=37^{\circ}20'44.36''$	$A=68^{\circ}24'30.92''$ $b=1246.9$ $c=785.92$ $R=647.7839$	$r=281.4538$ $r_1=1100.2722$ $r_2=1225.26$ $r_3=547.0578$
23. $a=187.035$ $A=84^{\circ}46'54.70''$ $B=61^{\circ}58'11.62''$	$C=33^{\circ}14'53.68''$ $b=165.783$ $c=102.972$ $R=93.90667$	$r=37.29846$ $r_1=208.0306$ $r_2=136.8516$ $r_3=68.04294$
24. $a=20077.67$ $A=82^{\circ}31'16.78''$ $C=32^{\circ}1'18.18''$	$B=65^{\circ}27'25.04''$ $b=18420.33$ $c=10737.33$ $R=10124.96$	$r=3982.967$ $r_1=21597.22$ $r_2=15821.55$ $r_3=7064.05$
25. $a=5618.156$ $B=71^{\circ}46'44.56''$ $C=54^{\circ}46'38.52''$	$A=53^{\circ}26'36.92''$ $b=6643.4$ $c=5713.6$ $R=3497.05$	$r=1696.249$ $r_1=4524.564$ $r_2=6503.416$ $r_3=4656.465$
26. $a=152.294$ $A=60^{\circ}58'15.38''$ $B=78^{\circ}51'16.72''$	$C=40^{\circ}10'27.9''$ $b=170.89$ $c=112.3631$ $R=87.08746$	$r=38.54804$ $r_1=128.2040$ $r_2=179.0552$ $r_3=79.63866$
27. $a=7710.144$ $B=51^{\circ}22'32.94''$ $C=63^{\circ}34'12.98''$	$A=65^{\circ}3'14.08''$ $b=6643.4$ $c=7614.7$ $R=4251.738$	$r=2087.92$ $r_1=7004.926$ $r_2=5283.446$ $r_3=6806.502$

EXAMPLES. V.—Continued.

DATA.	ANSWERS.		
28. $a=854.45$ $B=61^{\circ}45'17.76''$ $C=40^{\circ}45'14.78''$	$A=77^{\circ}29'27.46''$ $b=771.0144$ $c=571.36$ $R=437.6132$	$r=195.7687$ $r_1=881.4266$ $r_2=656.7996$ $r_3=407.9952$	
29. $a=1999.17$ $B=60^{\circ}46'54''$ $C=52^{\circ}6'26.74''$	$A=67^{\circ}3'39.26''$ $b=1894.38$ $c=1714.08$ $R=1085.274$	$r=533.2875$ $r_1=1857.858$ $r_2=1644.306$ $r_3=1372.217$	
30. $a=856.41$ $A=75^{\circ}13'54.42''$ $B=59^{\circ}17'26.38''$	$C=45^{\circ}28'39.2''$ $b=761.47$ $c=631.46$ $R=442.8341$	$r=206.7065$ $r_1=816.6095$ $r_2=640.079$ $r_3=471.3543$	

VI.—Triangles in which three sides are given.

First find r the radius of the inscribed circle by the formula $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$; then the three half angles from the equations $\tan \frac{1}{2}A = \frac{r}{s-a}$, $\tan \frac{1}{2}B = \frac{r}{s-b}$, $\tan \frac{1}{2}C = \frac{r}{s-c}$, correcting the results so as to make the sum of the three half angles exactly 90° . In general this will not involve alterations beyond $\pm .02''$; should greater changes be required, the work must be suspected. The values of r_1 , r_2 , r_3 may then be obtained from the equations $r_1 = s \tan \frac{1}{2}A$, $r_2 = s \tan \frac{1}{2}B$, $r_3 = s \tan \frac{1}{2}C$, and $R = \frac{1}{4}(r_1 + r_2 + r_3 - r)$. Should a further check be desired, $2R$ may be calculated from one of the three $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, $\frac{c}{\sin C}$, taking the angle nearest 45° . An example is worked out below.

DATA.		
$a=6634.39$	$s-a=1426.719$	$\log s-a=3.1543384$
$b=5646.67$	$s-b=2414.439$	$\log s-b=3.3828162$
$c=3841.158$	$s-c=4219.951$	$\log s-c=3.6253075$
$2s=16122.218$	$s=8061.109$	$-\log s=-3.9063948$
$s=8061.109$		$2 \log r=6.2560673$
		$\log r=3.1280337$

$L \tan \frac{1}{2}A$	$L \tan \frac{1}{2}B$	$L \tan \frac{1}{2}C$
13.1280337 3.1543384	13.1280337 3.3828162	13.1280337 3.6253075
9.9736953 43°16' - } 7073 2532 } 120 7200(2.84 2136 110	9.7452175 29°5' - } 403 2975 } 228 13680(4.6 1780	9.5027262 17°39' + } 6721 4371 } 541 32460(7.43 1863 115

$\frac{1}{2}A=43^{\circ}15'57.16''$	$\log r=3.1280337$	1.9736953	1.7452175	1.5027262
$\frac{1}{2}B=29^{\circ}45'54.40''$	113	3.9063948	3.9063948	3.9063948
$\frac{1}{2}C=17^{\circ}39'7.44''$	224	$\log r_1=3.8800901$	$\log r_2=3.6516123$	$\log r_3=3.4091210$
		873	075	2
$90^{\circ}0'0''$	<u>1342.87</u>	<u>7587.348</u>	<u>4483.449</u>	<u>2565.199</u>
		28	48	-2
$-r=-1342.87$	$\log c=3.5844621$	1784		
$r_1=7587.348$	$-\log \sin C=-1.7618208$	15=446	$A=86^{\circ}31'54.32''$	$r_1=7587.348$
$r_2=4483.449$	442	12=-3	$B=58^{\circ}9'50.8''$	$r_2=4483.449$
$r_3=2565.199$	$\log 2R=3.8225971$	30	$C=35^{\circ}18'14.88''$	$r_3=2565.199$
<u>$4R=13293.126$</u>	<u>6646.562</u>	41	$r=1342.87$	$A=3323.281$

EXAMPLES. VI.

DATA.

ANSWERS.

1. $a=159928.8$ $b=108792.1$ $c=183626.3$	$A=59^{\circ}59'59.8''$ $B=36^{\circ}5'39.6''$ $C=83^{\circ}54'20.6''$ $r=28246.42$	$r_1=130581.3$ $r_2=73694.15$ $r_3=203310.9$ $R=92334.98$	
2. $a=8175.801$ $b=9387.248$ $c=5561.611$	$A=60^{\circ}$ $B=83^{\circ}54'20.5''$ $C=36^{\circ}5'39.5''$ $r=1955.214$	$r_1=6675.515$ $r_2=10393.55$ $r_3=3767.335$ $R=4720.299$	$\Delta_2=3825.637$ $=b-c$
3. $a=9943.485$ $b=8660.254$ $c=5891.161$	$A=83^{\circ}54'20.5''$ $B=60^{\circ}$ $C=36^{\circ}5'39.5''$ $r=2071.068$	$r_1=11009.415$ $r_2=7071.068$ $r_3=3990.584$ $R=5000$	$\Delta_1=2071.067$ $\Delta_2=4052.325$ $\Delta_3=2071.067$
4. $a=28466.08$ $b=24792.47$ $c=16865.15$	$A=83^{\circ}54'20.4''$ $B=60^{\circ}$ $C=36^{\circ}5'39.6''$ $r=5929.031$	$r_1=31517.59$ $r_2=20242.97$ $r_3=11424.21$ $R=14313.93$	$\Delta_1=5929.008$ $\Delta_2=11600.93$ $\Delta_3=5929.019$
5. $a=2356.186$ $b=2365.324$ $c=1383.01$	$A=72^{\circ}36'20.57''$ $B=73^{\circ}19'46.76''$ $C=34^{\circ}3'52.67''$ $r=511.3707$	$r_1=2242.342$ $r_2=2272.171$ $r_3=935.0524$ $R=1234.549$	
6. $a=38714.69$ $b=33539.37$ $c=28471.26$	$A=76^{\circ}46'57.14''$ $B=57^{\circ}29'52.58''$ $C=45^{\circ}43'10.28''$ $r=9229.178$	$r_1=39904.455$ $r_2=27628.73$ $r_3=21232.35$ $R=19884.09$	
7. $a=468.2137$ $b=331.2970$ $c=258.9733$	$A=104^{\circ}18'19.36''$ $B=43^{\circ}17'8.40''$ $C=32^{\circ}24'32.24''$ $r=78.54318$	$r_1=681.13255$ $r_2=209.9995$ $r_3=153.8038$ $R=241.5982$	
8. $a=2846.608$ $b=2479.247$ $c=1687.515$	$A=83^{\circ}53'9.77''$ $B=59^{\circ}59'46.97''$ $C=36^{\circ}7'3.26''$ $r=593.149$	$r_1=3151.134$ $r_2=2024.438$ $r_3=1143.37$ $R=1431.446$	$\Delta_1=592.3827$ $\Delta_2=592.4747$ $\Delta_3=1159.2203$

EXAMPLES. VI.—Continued.

DATA.		ANSWERS.		
9.	$a=47674.27$ $b=46639.05$ $c=33611.96$	$A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$	$r_1=45438.86$ $r_2=42723.55$ $r_3=24385.79$ $R=25244.25$	$\Delta_1=7284.168$
10.	$a=466.3904$ $b=456.2141$ $c=270.8213$	$A=75^{\circ}0'0''$ $B=70^{\circ}52'57.7''$ $C=34^{\circ}7'2.3''$ $r=-100$	$r_1=457.8737$ $r_2=424.7102$ $r_3=183.1016$ $R=241.4214$	$\Delta_1=100$ $\Delta_2=159.8853$ $\Delta_3=60.32828$
11.	$a=19674.04$ $b=15361.02$ $c=9837.02$	$A=100^{\circ}21'30.58''$ $B=50^{\circ}10'45.12''$ $C=29^{\circ}27'44.3''$ $r=3312.615$	$r_1=26908.75$ $r_2=10504.845$ $r_3=5899.026$ $R=10000$	
12.	$a=9682.458$ $b=9990.51$ $c=2919.436$	$A=75^{\circ}31'20.68''$ $B=87^{\circ}30'11.67''$ $C=16^{\circ}58'27.65''$ $r=1250$	$r_1=8749.988$ $r_2=10814.376$ $r_3=1685.644$ $R=5000$	
13.	$a=47674.27$ $b=46639.05$ $c=33611.96$	$A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$	$r_1=45438.76$ $r_2=42723.55$ $r_3=24385.78$ $R=25244.25$	
14.	$a=375.3259$ $b=312.7715$ $c=271.0686$	$A=79^{\circ}39'36.48''$ $B=55^{\circ}3'53.24''$ $C=45^{\circ}16'30.28''$ $r=86.95647$	$r_1=400$ $r_2=250$ $r_3=200$ $R=190.7608$	
15.	$a=14538.87$ $b=34645.37$ $c=30139.84$	$A=24^{\circ}42'1.5''$ $B=95^{\circ}16'15.9''$ $C=60^{\circ}1'42.6''$ $r=5500.802$	$r_1=8684.138$ $r_2=43489.62$ $r_3=22912.04$ $R=17396.25$	
16.	$a=7406.25$ $b=31560.45$ $c=30137.34$	$A=13^{\circ}32'3.74''$ $B=94^{\circ}13'58.62''$ $C=72^{\circ}13'57.64''$ $r=3221.169$	$r_1=4100.008$ $r_2=37203.835$ $r_3=25210.84$ $R=15823.38$	
17.	$a=10927.86$ $b=8517.08$ $c=3642.62$	$A=122^{\circ}47'45.58''$ $B=40^{\circ}55'54.4''$ $C=16^{\circ}16'20.02''$ $r=1129.583$	$r_1=21171.03$ $r_2=4308.21$ $r_3=1650.345$ $R=6500$	
18.	$a=5761.367$ $b=5017.848$ $c=3413.403$	$A=83^{\circ}54'20.57''$ $B=60^{\circ}0'0''$ $C=36^{\circ}5'39.43''$ $r=1200$	$r_1=6378.981$ $r_2=4097.055$ $r_3=2312.188$ $R=2897.056$	$\Delta_1=\Delta_3=r$ $\Delta_2=2347.964$ $=a-c$
19.	$a=94371.5$ $b=124351.6$ $c=106381.7$	$A=47^{\circ}30'$ $B=76^{\circ}17'14.52''$ $C=56^{\circ}12'45.48''$ $r=30000.31$	$r_1=71524.76$ $r_2=127657.63$ $r_3=86817.93$ $R=64000$	$\Delta_1=15998.76$
20.	$a=4157.855$ $b=4812.793$ $c=3608.528$	$A=57^{\circ}2'28''$ $B=76^{\circ}13'21.6''$ $C=46^{\circ}44'10.4''$ $r=1158.427$	$r_1=3417.89$ $r_2=4933.676$ $r_3=2717.593$ $R=2477.683$	

EXAMPLES. VI.—Continued.

DATA.		ANSWERS.		
21.	$a=9401.45$ $b=7360.12$ $c=5318.79$	The sides in $A.P.$, and the greatest angle exceeds the least by 60° . $8r=3R$.	$A=94^\circ 20' 27.92''$ $B=51^\circ 19' 4.16''$ $C=34^\circ 20' 27.92''$ $r=1767.844$	$r_1=11910.02$ $r_2=5303.53$ $r_3=3411.289$ $R=4714.249$ $\Delta_1=2357.12$
22.	$a=26925.38$ $b=18604.98$ $c=10284.58$		$A=135^\circ 31' 20.94''$ $B=28^\circ 57' 18.08''$ $C=15^\circ 31' 20.98''$ $r=240189.2$	$r_1=6125317$ $r_2=720567.9$ $r_3=380361.9$ $R=1921514$ $\Delta_1=1664081$
23.	$a=964.701$ $b=955.078$ $c=945.455$	The sides in $A.P.$, $A-C=2^\circ$.	$A=61^\circ 0' 18.14''$ $B=59^\circ 59' 23.70''$ $C=59^\circ 0' 18.10''$ $r=275.6513$	$r_1=843.9606$ $r_2=826.9536$ $r_3=810.6189$ $R=551.4705$ $\Delta_1=9.622460$
24.	$a=2089.88$ $b=2413.68$ $c=1206.96$		$A=59^\circ 58' 46.83''$ $B=90^\circ 1' 1.35''$ $C=30^\circ 0' 11.82''$ $r=-441.7114$	$r_1=1647.81$ $r_2=2856.109$ $r_3=765.1524$ $R=1206.84$ $\Delta_1=624.7480$ $\Delta_2=1208.060$ $\Delta_3=625.855$
25.	$a=4794.217$ $b=4329.393$ $c=3655.232$		$A=73^\circ 15' 10.41''$ $B=59^\circ 51' 12.13''$ $C=46^\circ 53' 37.46''$ $r=1185.846$	$r_1=4749.779$ $r_2=3678.041$ $r_3=2771.157$ $R=2503.283$
26.	$a=544.36$ $b=426.39$ $c=425.09$		$A=79^\circ 28' 50.42''$ $B=50^\circ 21' 53.56''$ $C=50^\circ 9' 16.02''$ $r=127.6707$	$r_1=580.2548$ $r_2=328.155$ $r_3=326.5914$ $R=276.8326$
27.	$a=51509.95$ $b=72027.91$ $c=63926$		$A=44^\circ 1' 38.62''$ $B=76^\circ 22' 16.08''$ $C=59^\circ 36' 5.3''$ $r=17070.53$	$r_1=37896.22$ $r_2=73721.52$ $r_3=53682.39$ $R=37057.2$
28.	$a=10301.99$ $b=10825.782$ $c=9205.4$		$A=61^\circ 16' 11.50''$ $B=67^\circ 8' 35.18''$ $C=51^\circ 35' 13.32''$ $r=2880.918$	$r_1=8981.956$ $r_2=10065.804$ $r_3=7329.696$ $R=5874.135$
29.	$a=5844.305$ $b=7202.791$ $c=4602.7$		$A=54^\circ 4' 4.86''$ $B=86^\circ 18' 43.14''$ $C=39^\circ 37' 12''$ $r=1520.916$	$r_1=4503.11$ $r_2=8274.385$ $r_3=3178.898$ $R=3608.869$
30.	$a=1168.861$ $b=1082.5782$ $c=1278.52$		$A=58^\circ 38' 46.14''$ $B=52^\circ 16' 21.68''$ $C=69^\circ 4' 52.18''$ $r=334.8419$	$r_1=991.3952$ $r_2=866.0432$ $r_3=1214.8782$ $R=684.3687$
31.	$a=1262.92$ $b=1332.60$ $c=1142.72$		$A=60^\circ 46' 54.01''$ $B=67^\circ 3' 39.26''$ $C=52^\circ 9' 26.73''$ $r=355.525$	$r_1=1096.2042$ $r_2=1238.5722$ $r_3=914.8112$ $R=723.5157$
32.	$a=18943.8$ $b=25692.3$ $c=22844.1$		$A=45^\circ 28' 39.20''$ $B=75^\circ 13' 54.42''$ $C=59^\circ 17' 26.38''$ $r=6201.195$	$r_1=14140.63$ $r_2=25998.28$ $r_3=19202.37$ $R=13285.02$

EXAMPLES. VI.—Continued.

DATA.

ANSWERS.

3.	$a=70137$ $b=66630$ $c=76147$	$A=58^{\circ}23'20.49''$ $B=54^{\circ}0'9.25''$ $C=71^{\circ}36'30.26''$ $r=20293.99$	$r_1=59483.42$ $r_2=54245.56$ $r_3=71278.06$ $R=41178.26$	
1.	$a=3506.8$ $b=4282.05$ $c=2856.8$	$A=54^{\circ}31'24.61''$ $B=83^{\circ}55'0.40''$ $C=41^{\circ}33'34.99''$ $r=935.773$	$r_1=2742.83$ $r_2=4785.72$ $r_3=2019.82$ $R=2153.15$	
5.	$a=786.934$ $b=604.81$ $c=431.5957$	$A=97^{\circ}23'42.2''$ $B=49^{\circ}39'21.88''$ $C=32^{\circ}56'55.92''$ $r=141.9714$	$r_1=1037.641$ $r_2=421.7921$ $r_3=269.6064$ $R=396.767$	
3.	$a=786.934$ $b=748.887$ $c=431.5957$	$A=78^{\circ}35'37.44''$ $B=68^{\circ}53'4.20''$ $C=32^{\circ}31'18.36''$ $r=161.04$	$r_1=805.0664$ $r_2=674.625$ $r_3=286.9277$ $R=401.3948$	
1.	$a=43783$ $b=43397$ $c=43012$	$A=60^{\circ}53'9.3''$ $B=59^{\circ}59'29.07''$ $C=59^{\circ}7'21.63''$ $r=12525.76$	$r_1=3857.23$ $r_2=37576.69$ $r_3=36921.59$ $R=25057.44$	$\Delta_1=385.1494$ $\Delta_2=771.168$ $\Delta_3=399.6372$
3.	$a=96.264$ $b=114.735$ $c=106.848$	$A=51^{\circ}20'53.50''$ $B=68^{\circ}33'38.16''$ $C=60^{\circ}5'28.34''$ $r=30.12111$	$r_1=76.39623$ $r_2=108.3302$ $r_3=91.92327$ $R=61.63215$	$\Delta_1=9.255574$
2.	$a=224.626$ $b=218.323$ $c=199.92$	$A=64^{\circ}47'27.34''$ $B=61^{\circ}34'17.14''$ $C=53^{\circ}38'15.52''$ $r=61.428976$	$r_1=203.9497$ $r_2=191.5018$ $r_3=162.4983$ $R=124.1302$	
2.	$a=14724$ $b=27601$ $c=17224$	$A=50^{\circ}0'30.76''$ $B=66^{\circ}19'31.38''$ $C=63^{\circ}39'57.86''$ $r=4687.54$	$r_1=11554.79$ $r_2=16188.95$ $r_3=15380.63$ $R=9609.21$	
1.	$a=63341$ $b=52587$ $c=29448$	$A=97^{\circ}2'17.90''$ $B=55^{\circ}29'1.08''$ $C=27^{\circ}28'41.02''$ $r=10571.92$	$r_1=82214.21$ $r_2=38229.75$ $r_3=17771.81$ $R=31910.96$	
2.	$a=1266.82$ $b=704.04$ $c=1166.82$	$A=81^{\circ}9'58.50''$ $B=33^{\circ}18'34.24''$ $C=65^{\circ}31'27.26''$ $r=258.7082$	$r_1=1343.8572$ $r_2=469.3244$ $r_3=1009.5812$ $R=641.0137$	
3.	$a=1750.23$ $b=1577.61$ $c=1033.44$	$A=81^{\circ}17'39.16''$ $B=62^{\circ}59'51.52''$ $C=35^{\circ}42'29.32''$ $r=369.5202$	$r_1=1872.1467$ $r_2=1336.2372$ $r_3=702.3975$ $R=885.3153$	
1.	$a=13467.87$ $b=8287.92$ $c=7251.93$	$A=120^{\circ}$ $B=32^{\circ}12'15.22''$ $C=27^{\circ}47'44.78''$ $r=1794.387$	$r_1=25121.42$ $r_2=4186.903$ $r_3=3588.774$ $R=7775.68$	$\Delta_2=15539.85$ $=b+c$

VII.—Triangles in which two sides and the included angle are given.

Let b, c be the two given lengths ($b > c$), A the included angle, then we find $\frac{1}{2}(B - C)$ by the equation $\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \cot \frac{1}{2}A$; and $\frac{1}{2}(B + C) = 90^\circ - \frac{A}{2}$; $B = \frac{1}{2}(B + C) + \frac{1}{2}(B - C)$, $C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$. The value of $2R$ is then calculated from each of the expressions $b/\sin B$, $c/\sin C$, which should agree (*q.p.*). The formulae $a = 2R \sin A$, $r = (s - a) \tan \frac{A}{2}$, $r_1 = s \tan \frac{A}{2}$, $r_2 = (s - c) \cot \frac{1}{2}A$, $r_3 = (s - b) \cot \frac{1}{2}A$ enable us to find a, r, r_1, r_2, r_3 ; and the equation $4R = r_1 + r_2 + r_3 - r$ supplies another check.

DATA.		EXAMPLE WORKED OUT.	
$b=33417$	$b-c=9990$	$L \cot \frac{1}{2}A = 10.5228579$	4592
$c=23427$	$b+c=56844$	$\log(b-c) = 3.9995655$	$10' = 765 \quad 3$
$A=33^\circ 23' 37''$	$\frac{A}{2} = 16^\circ 41' 48.5''$	$-\log(b+c) = -4.7546846$	$1.5 = 114 \quad 8$
		$L \tan \frac{1}{2}(B - C) = 9.7678268$	4560 (1.58
		$\left. \begin{array}{l} 50^\circ 22' - \\ 2896 \end{array} \right\}$	$\frac{344}{76} \quad \frac{1664}{216}$
$\frac{1}{2}(B - C) = 30^\circ 21' 58.42''$		$\log b = 4.5239675$	$308 = 5 + 1\frac{1}{8} + \frac{1}{8}$
$\frac{1}{2}(B + C) = 73^\circ 18' 11.5''$		$-\log \sin B = -1.9874955$	$250 \quad 4$
$B = 103^\circ 40' 9.92''$	$180^\circ - B = 76^\circ 19' 50.08''$	$\log 2R = 4.5364463$	$5 \quad 7$
$C = 42^\circ 56' 13.08''$		$\frac{48}{15}$	
$\log c = 4.3697167$	1358	$\log 2R = 4.5364463$	1916
$-\log \sin C = 1.8332408$	$\left. \begin{array}{l} 12 = 271 \quad 6 \\ 1.2 = 27 \quad 2 \\ - .12 = -2 \quad 7 \end{array} \right\}$	$\log \sin A = 1.7405505$	$36 = 1149 \quad 6$
$\log 2R = 4.5364463$		$\log a = 4.2771149$	$1 = 31 \quad 9$
		$\frac{047}{102}$	
$\log \cot \frac{1}{2}A = 0.5229459$	$a = 18928.44$	$\log(s-a) = 4.2777877$	
$\log \tan \frac{1}{2}A = 1.4770541$	$b = 33417$	$\log(s-b) = 3.6502317$	
	$c = 23427$	$\log(s-c) = 4.1601449$	
	75772.44	$\log s = 4.5784813$	
	$s = 37886.22$		
$\log r$	$\log r_1$	$\log r_2$	$\log r_3$
1.4770541	1.4770541	$.5229459$	$.5229459$
4.2777877	4.5784813	4.1601449	3.6502317
3.7548418	4.0555354	4.6830908	4.1731776
374	12	831	571
5686.457	11364.11	48204.86	11899.70
44	42	77	250
			$4R = 68782.22$

ANSWERS.

$B = 103^\circ 40' 9.92''$	$a = 18928.44$	$r = 5686.457$	$r_2 = 48204.86$
$C = 42^\circ 56' 13.08''$	$R = 17195.56$	$r_1 = 11364.11$	$r_3 = 14899.70$

EXAMPLES. VII.

DATA.		ANSWERS.	
$b=23571.26$ $c=20753.82$ $A=67^{\circ}31'55''$		$B=61^{\circ}39'53.64''$ $C=50^{\circ}48'11.36''$ $a=24747.05$ $K=13389.91$	$r=6544.85$ $r_1=23090.21$ $r_2=20614.16$ $r_3=16400.11$
$b=8438.715$ $c=4219.357$ $A=59^{\circ}37'56.4''$		$B=90^{\circ}22'6.07''$ $C=29^{\circ}59'57.53''$ $a=7281.067$ $K=4219.444$	$r=1540.727$ $r_1=5713.363$ $r_2=10033.872$ $r_3=2671.274$
$b=378.9217$ $c=269.1351$ $A=88^{\circ}59'2.4''$		$B=55^{\circ}17'30.72''$ $C=35^{\circ}43'26.88''$ $a=460.8674$ $K=230.4699$	$r=91.94954$ $r_1=544.71625$ $r_2=290.4319$ $r_3=178.6812$
$b=13347.61$ $c=11258.73$ $A=47^{\circ}31'57''$		$B=77^{\circ}8'44''$ $C=55^{\circ}19'19''$ $a=10099.1$ $K=6845.362$	$r=3194.125$ $r_1=7641.255$ $r_2=13839$ $r_3=9095.308$
$b=99144.49$ $c=60876.14$ $A=45^{\circ}$		$B=97^{\circ}30'$ $C=37^{\circ}30'$ $a=70710.68$ $K=50000$	$r=18496.69$ $r_1=47786.01$ $r_2=131549.34$ $r_3=39161.36$
$b=5.6794$ $c=5.6318$ $A=44^{\circ}59'6.26''$		$B=68^{\circ}5'23.27''$ $C=66^{\circ}55'30.47''$ $a=4.327479$ $K=3.060782$	$r=1.445844$ $r_1=3.237680$ $r_2=5.283127$ $r_3=5.168167$
$b=87.92154$ $c=29.30718$ $A=70^{\circ}31'43.62''$		$B=90^{\circ}$ $C=19^{\circ}28'16.38''$ $a=82.89322$ $K=43.96077$	$r=12.13143$ $r_1=70.75379$ $r_2=100.06097$ $r_3=17.16775$
$b=99905.1$ $c=29194.36$ $A=75^{\circ}31'20.7''$		$B=87^{\circ}30'11.65''$ $C=16^{\circ}58'27.65''$ $a=96824.54$ $K=50000$	$r=12500$ $r_1=87499.9$ $r_2=108143.67$ $r_3=16856.4$
$b=1243.516$ $c=1063.817$ $A=47^{\circ}30'$		$B=76^{\circ}17'14.56''$ $C=56^{\circ}12'45.44''$ $a=943.715$ $K=640$	$r=300.0031$ $r_1=715.2478$ $r_2=1276.5762$ $r_3=868.1792$
$b=57777.78$ $c=47222.22$ $A=129^{\circ}18'27.52''$	$180r=9r_1=76r_2=95r_3$	$B=28^{\circ}4'21.01''$ $C=22^{\circ}37'11.47''$ $a=95000$ $K=61388.89$	$r=10555.56$ $r_1=211111.1$ $r_2=25000$ $r_3=20000$
$b=5237.828$ $c=6673.472$ $A=87^{\circ}50'57.8''$		$B=38^{\circ}56'32.8''$ $C=53^{\circ}12'29.4''$ $a=8327.466$ $K=4166.667$	$r=1725.888$ $r_1=9746.503$ $r_2=3577.745$ $r_3=5068.313$
$b=14045.86$ $c=8109.38$ $A=30^{\circ}$		$B=120^{\circ}$ $C=30^{\circ}$ $a=8109.38$ $K=8109.38$	$r=1881.79$ $r_1=4054.69$ $r_2=26209.93$ $r_3=4054.69$

$\Delta_1=159.9976$
 $\Delta_2=343.1666$
 $\Delta_3=185.617$

$\Delta_1=1725.894$

EXAMPLES. VII.—Continued.

DATA.		ANSWERS.		
13.	$b=1404.59$ $c=810.94$ $A=30^{\circ}0'0.07''$	$B=119^{\circ}59'59.965''$ $C=29^{\circ}59'59.965''$ $a=810.941$ $R=810.9405$	$r=188.1794$ $r_1=405.4705$ $r_2=2621.0007$ $r_3=405.4702$	
14.	$b=4568.39$ $c=864.29$ $A=110^{\circ}34'11.96''$	$B=60^{\circ}$ $C=9^{\circ}25'48.04''$ $a=4938.8$ $R=2637.561$	$r=356.427$ $r_1=7484.971$ $r_2=2993.989$ $r_3=427.712$	$\Delta_1=2253.11$ $\Delta_2=4074.51$ $\Delta_3=2253.11$
15.	$b=9237.97$ $c=6598.55$ $A=81^{\circ}47'12.46''$	$B=60^{\circ}$ $C=38^{\circ}12'47.54''$ $a=10557.08$ $R=5333.545$	$r=2285.805$ $r_1=11429.025$ $r_2=7614.35$ $r_3=4571.61$	$\Delta_1=2015.85$ $\Delta_2=3959.13$ $\Delta_3=2015.85$
16.	$b=47898.32$ $c=39817.65$ $A=65^{\circ}26'23.6''$	$B=65^{\circ}26'23.6''$ $C=49^{\circ}7'12.8''$ $a=47898.32$ $R=26331.48$	$r=12791.04$ $r_1=43564.76$ $r_2=43564.76$ $r_3=30987.44$	
17.	$b=48127.93$ $c=36085.28$ $A=57^{\circ}2'28''$	$B=76^{\circ}13'21.62''$ $C=46^{\circ}44'10.38''$ $a=41578.55$ $R=24776.83$	$r=11584.27$ $r_1=34178.9$ $r_2=49336.76$ $r_3=27175.93$	
18.	$b=4157.855$ $c=3608.528$ $A=76^{\circ}13'21.6''$	$B=57^{\circ}2'28''$ $C=46^{\circ}44'10.4''$ $a=4812.793$ $R=2477.683$	$r=1158.427$ $r_1=4933.676$ $r_2=3417.89$ $r_3=2717.593$	
19.	$b=23571.26$ $c=20753.52$ $A=67^{\circ}31'55''$	$B=61^{\circ}39'55.86''$ $C=50^{\circ}48'9.14''$ $a=24746.85$ $R=13389.84$	$r=6544.721$ $r_1=23090.03$ $r_2=20614.22$ $r_3=16399.73$	
20.	$b=30901.7$ $c=58778.53$ $A=18^{\circ}$	$B=18^{\circ}$ $C=144^{\circ}$ $a=30901.7$ $R=50000$	$r=4654.803$ $r_1=9549.152$ $r_2=9549.152$ $r_3=185556.5$	
21.	$b=12369.12$ $c=4123.04$ $A=40^{\circ}55'55''$	The triangle in which the greatest side is 3 times the least, and the greatest angle 3 times the mean. $q.p.$ $B=122^{\circ}47'44.84''$ $C=16^{\circ}16'20.16''$ $a=9640.399$ $R=7357.262$	$r=1278.566$ $r_1=4876.439$ $r_2=23963.16$ $r_3=1868.021$	$\Delta_1=5942.7$ $\Delta_2=14974.3$ $\Delta_3=9397.9$
22.	$b=13571$ $c=8965$ $A=79^{\circ}55'58.5''$	$B=63^{\circ}44'18.57''$ $C=36^{\circ}19'42.93''$ $a=14899.98$ $R=7566.486$	$r=3199.883$ $r_1=15687.6$ $r_2=11636.99$ $r_3=6141.241$	$\Delta_1=2971.19$ $\Delta_2=6021.61$ $\Delta_3=3159.60$
23.	$b=18791.69$ $c=11275.05$ $A=68^{\circ}35'57.6''$	$B=75^{\circ}49'39.51''$ $C=35^{\circ}34'22.89''$ $a=18045.27$ $R=9690.8$	$r=4100.197$ $r_1=16409.691$ $r_2=18736.413$ $r_3=7717.292$	$\Delta_1=3800.41$
24.	$b=133.7127$ $c=206.3273$ $A=58^{\circ}21'37.4''$	$B=39^{\circ}53'33.16''$ $C=81^{\circ}44'49.44''$ $a=177.4979$ $R=104.2432$	$r=45.384$ $r_1=144.5037$ $r_2=93.9095$ $r_3=223.9436$	

EXAMPLES. VII.—Continued.

DATA.		ANSWERS.	
25. $b=8233.59$ $c=6618.853$ $A=83^{\circ}8'1.8''$		$B=55^{\circ}25'20.65''$ $C=41^{\circ}26'37.55''$ $a=9928.291$ $R=5000.006$	$r=2183.392$ $r_1=10987.9$ $r_2=6508.179$ $r_3=4687.339$
26. $b=23870.56$ $c=10775.56$ $A=41^{\circ}24'34.6''$	$b+c=2a, B-C=90^{\circ}$ $R=4r, r_1=3r$	$B=114^{\circ}17'42.7''$ $C=24^{\circ}17'42.7''$ $a=17323.06$ $R=13095$	$r=3273.749$ $r_1=9821.247$ $r_2=40239.337$ $r_3=5593.201$
27. $b=5361.796$ $c=5957.551$ $A=23^{\circ}33'23.3''$		$B=64^{\circ}3'19.87''$ $C=92^{\circ}23'10.83''$ $a=2383.022$ $R=2981.364$	$r=931.6764$ $r_1=1428.571$ $r_2=4285.715$ $r_3=7142.852$
28. $b=138.74$ $c=116.82$ $A=34^{\circ}33'37''$		$B=88^{\circ}8'4.07''$ $C=57^{\circ}18'18.93''$ $a=78.7452$ $R=69.4068$	$r=27.50223$ $r_1=51.99869$ $r_2=161.7969$ $r_3=91.33386$
29. $b=1687.48$ $c=2531.22$ $A=104^{\circ}28'39.04''$		$B=28^{\circ}57'18.08''$ $C=46^{\circ}34'2.88''$ $a=3374.96$ $R=1742.822$	$r=544.6318$ $r_1=4901.688$ $r_2=980.3372$ $r_3=1633.896$ $\Delta_1=1067.256$ $\Delta_2=2586.858$ $\Delta_3=1555.78$
30. $b=31428.54$ $c=35714.25$ $A=69^{\circ}47'25.64''$	$74r=20r_1=30r_2=24r_3$	$B=49^{\circ}52'33.45''$ $C=60.20^{\circ}0.91''$ $a=38571.41$ $R=20550.91$	$r=9964.067$ $r_1=36867.08$ $r_2=24578.06$ $r_3=30722.57$
31. $b=273.8163$ $c=262.1835$ $A=8^{\circ}13'57.6''$		$B=102^{\circ}39'55.14''$ $C=69^{\circ}6'7.26''$ $a=40.18648$ $R=140.323$	$r=17.8412$ $r_1=20.73332$ $r_2=360.02$ $r_3=198.3801$
32. $b=24.1368$ $c=12.0696$ $A=59^{\circ}58'48''$		$B=90^{\circ}1'0.17''$ $C=30^{\circ}0'11.83''$ $a=20.89888$ $R=12.068405$	$r=4.417119$ $r_1=16.47823$ $r_2=28.56097$ $r_3=7.651542$ $\Delta_1=6.247486$ $\Delta_2=12.08062$ $\Delta_3=6.25855$
33. $b=856.41$ $c=761.47$ $A=45^{\circ}28'39.2''$		$B=75^{\circ}13'54.53''$ $C=59^{\circ}17'26.27''$ $a=631.4601$ $R=442.834$	$r=206.7065$ $r_1=471.3542$ $r_2=866.6096$ $r_3=640.079$
4. $b=786.934$ $c=748.887$ $A=32^{\circ}31'18.36''$		$B=78^{\circ}35'37.44''$ $C=68^{\circ}53'4.20''$ $a=431.5957$ $R=401.3948$	$r=161.04$ $r_1=286.9277$ $r_2=805.0664$ $r_3=674.6250$
5. $b=3734.67$ $c=3593.19$ $A=52^{\circ}31'49.2''$		$B=65^{\circ}58'31.37''$ $C=61^{\circ}29'39.43''$ $a=3245.25$ $R=2044.444$	$r=1007.332$ $r_1=2608.78$ $r_2=3431.515$ $r_3=3144.813$ $\Delta_1=246.7459$ $\Delta_2=489.2463$ $\Delta_3=243.3927$
16. $b=487.139$ $c=479.363$ $A=48^{\circ}39'39.6''$		$B=66^{\circ}41'20''$ $C=64^{\circ}36'0.4''$ $a=398.2614$ $R=265.2195$	$r=128.4674$ $r_1=308.5446$ $r_2=448.9992$ $r_3=431.8017$ $\Delta_1=46.87498$ $\Delta_2=85.8304$ $\Delta_3=38.99333$

EXAMPLES. VII.—Continued.

DATA.	ANSWERS.
37. $b=433.97$ $c=430.12$ $A=60^{\circ}53'9.3''$	$B=59^{\circ}59'29.06''$ $C=59^{\circ}7'21.64''$ $a=437.83$ $R=250.5744$ $r=125.2576$ $r_1=382.5723$ $r_2=375.7669$ $r_3=369.2159$ $\Delta_1=3.851494$ $\Delta_2=3.996372$ $\Delta_3=7.71168$
38. $b=1057.81$ $c=1220.55$ $A=27^{\circ}47'44.76''$	$B=60^{\circ}0'0''$ $C=92^{\circ}12'15.24''$ $a=569.59$ $R=610.7271$ $r=211.4054$ $r_1=352.3423$ $r_2=822.1324$ $r_3=1479.839$ $\Delta_1=338.7705$ $\Delta_2=650.9600$ $=c-a$ $\Delta_3=338.7705$

VIII.—Triangles in which two sides and the angle opposite the greater are given.

Let a, b be the two given lengths ($a > b$), and A the given angle. First calculate R by the equation $2R = a/\sin A$; B must be an acute angle and is found from $\sin B = b/2R$; C from $A + B + C = 180^{\circ}$; c from $c = 2R \sin C$. Calculate r, r_1, r_2, r_3 exactly as in V.

EXAMPLE WORKED OUT.

DATA. $a=4812.793$ $b=3608.528$ $A=76^{\circ}13'21.6''$	$\log a=3.6823972$ $-\log \sin A=1.9873103$ $\log 2R=3.6950757$ 4955.366 57	310 $18=93$ $3.6=18 \ 6$ $101 \ 6$	$\log b=3.5573301$ $-\log 2R=3.6950757$ $L \sin B=9.8622544$ $46^{\circ}44' +$ $1188 \ } \ 338$ 206	$12360(10.4''$ 480			
$A=76^{\circ}13'21.6''$ $B=46^{\circ}44'10.4''$ $C=57^{\circ}2'28.0''$	$\log 2R=3.6950757$ $\log \sin C=1.9237554$ $\log c=3.6188693$ 4157.855 57	819 $24=327 \ 6$ $4=54 \ 6$	$\frac{A}{2}=38^{\circ}6'40.8''$ $\log \tan \frac{A}{2}=1.8943715$ 1769 $=1.8945484$ $\log \cot \frac{A}{2}=0.1054516$	2602 $36=1561 \ 2$ $4.8=208 \ 2$			
$a=4812.793$ $b=3608.528$ $c=4157.855$ $2s=12579.176$ $s=6289.588$	$s-a=1476.795$ $s-b=2681.06$ $s-c=2131.733$ $s=6289.588$	$\log=3.1693202$ $\log=3.4283065$ $\log=3.3287329$ $\log=3.7986222$	$\log r$ 3.1693202 1.8945484 3.0638686 585 1158.427 101	$\log r_1$ 3.7986222 1.8945484 3.6931706 639 4933.676 67	$\log r_2$ 3.3287329 $.1054516$ 3.4341845 696 2717.593 149	$\log r_3$ 3.4283065 $.1054516$ 3.5337581 466 3417.89 115	$-r=1158.427$ $r_1=4933.676$ $r_2=2717.593$ $r_3=3417.89$ $4R=9910.732$

ANSWERS.

$R=2477.683$	$C=57^{\circ}2'28''$	$r=1158.427$	$r_2=2717.593$
$B=46^{\circ}44'10.4''$	$c=4157.855$	$r_1=4933.676$	$r_3=3417.89$

EXAMPLES VIII.

DATA.

ANSWERS.

1. $a=266.0007$ $b=138.7214$ $A=62^{\circ}27'27.4''$		$R=150$ $B=27^{\circ}32'32.6''$ $C=90^{\circ}$ $c=300$	$r=52.36105$ $r_1=213.63965$ $r_2=86.36035$ $r_3=352.36105$	$\Delta_1=82.41167$
2. $a=2620.87$ $b=1513.16$ $A=120^{\circ}$		$R=1513.16$ $B=30^{\circ}$ $C=30^{\circ}$ $c=1513.16$	$r=351.13$ $r_1=4890.611$ $r_2=756.5801$ $r_3=756.5801$	$\Delta_1=1107.84$
3. $a=82893.16$ $b=29307.18$ $A=70^{\circ}31'43.55''$		$R=43960.735$ $B=19^{\circ}28'16.45''$ $C=90^{\circ}$ $c=87921.47$	$r=12139.435$ $r_1=70753.725$ $r_2=17167.745$ $r_3=100060.905$	$\Delta_1=29414.78$
4. $a=2731.843$ $b=1600.277$ $A=72^{\circ}58'7.5''$		$R=14285.715$ $B=34^{\circ}3'45''$ $C=72^{\circ}58'7.5''$ $c=2731.843$	$r=591.7336$ $r_1=2612.039$ $r_2=1081.942$ $r_3=2612.039$	$\Delta_1=591.7373$
5. $a=27442.1$ $b=11366.89$ $A=67^{\circ}30'$		$R=14851.56$ $B=22^{\circ}30'$ $C=90^{\circ}$ $c=29703.12$	$r=4552.935$ $r_1=22889.165$ $r_2=6813.955$ $r_3=34256.055$	$\Delta_1=11629.12$
6. $a=968.2454$ $b=291.9436$ $A=75^{\circ}31'20.68''$		$R=500$ $B=16^{\circ}58'27.68''$ $C=87^{\circ}30'11.64''$ $c=999.05114$	$r=125.0001$ $r_1=874.9987$ $r_2=168.564$ $r_3=1081.4372$	$\Delta_1=353.5534$
7. $a=24150.16$ $b=22202.21$ $A=57^{\circ}41'57.5''$		$R=14285.72$ $B=50^{\circ}59'37.48''$ $C=71^{\circ}18'25.01''$ $c=27064.26$	$r=6918.089$ $r_1=20222.015$ $r_2=17506.506$ $r_3=26332.425$	$\Delta_1=2534.167$
8. $a=8571.427$ $b=7142.856$ $A=73^{\circ}44'23.3''$		$R=4464.286$ $B=53^{\circ}7'48.35''$ $C=53^{\circ}7'48.35''$ $c=7142.856$	$r=2412.857$ $r_1=8571.427$ $r_2=5714.284$ $r_3=5714.284$	$\Delta_1=892.8586$
9. $a=673.1282$ $b=336.5642$ $A=92^{\circ}27'53.4''$		$R=336.8757$ $B=29^{\circ}53'36.39''$ $C=57^{\circ}38'30.21''$ $c=569.13$	$r=121.3962$ $r_1=824.1232$ $r_2=211.003$ $r_3=433.7731$	
10. $a=47124.88$ $b=35623.91$ $A=65^{\circ}37'48.6''$		$R=25867.18$ $B=43^{\circ}31'7.76''$ $C=70^{\circ}51'3.64''$ $c=48871.84$	$r=12048.9$ $r_1=42436.38$ $r_2=26268.18$ $r_3=46813.06$	
11. $a=9401.45$ $b=7360.12$ $A=94^{\circ}20'27.9''$	$a+c=2b$ $A-C=60^{\circ}$	$R=4714.249$ $B=51^{\circ}19'4.12''$ $C=34^{\circ}20'27.98''$ $c=5318.792$	$r=1767.845$ $r_1=11910.022$ $r_2=5303.53$ $r_3=3411.291$	$\Delta_1=2357.121$
12. $a=7791.368$ $b=5387.123$ $A=69^{\circ}31'48.12''$		$R=4158.254$ $B=40^{\circ}12'23.22''$ $C=70^{\circ}15'48.66''$ $c=7827.959$	$r=1882.335$ $r_1=7290.425$ $r_2=3854.223$ $r_3=7370.705$	

EXAMPLES VIII.—Continued.

DATA.		ANSWERS.	
25.	$a=11878.92$ $b=10630.44$ $A=63^{\circ}27'15.2''$	$R=6639.403$ $B=53^{\circ}10'59.98''$ $C=63^{\circ}21'44.82''$ $c=11869.4$	$r=3283.28$ $r_1=10627.62$ $r_2=8604.674$ $r_3=10608.6$
26.	$a=8129.16$ $b=9246.77$ $B=63^{\circ}57'31.52''$	$R=5145.795$ $A=52^{\circ}10'28.76''$ $C=63^{\circ}51'59.72''$ $c=9239.49$	$r=2535.525$ $r_1=6515.731$ $r_2=8308.923$ $r_3=8294.053$
27.	$a=8704.287$ $b=7895.319$ $A=67^{\circ}34'13.06''$	$R=4708.338$ $B=56^{\circ}58'32.84''$ $C=55^{\circ}27'14.1''$ $c=7756.236$	$r=2324.09$ $r_1=8147.836$ $r_2=6608.74$ $r_3=6400.864$
28.	$a=2777.331$ $b=2028.932$ $A=81^{\circ}2'39.88''$	$R=1405.803$ $B=46^{\circ}11'20.42''$ $C=52^{\circ}45'59.7''$ $c=2238.537$	$r=636.8488$ $r_1=3010.775$ $r_2=1502.031$ $r_3=1747.254$
29.	$a=6004.551$ $b=4940.314$ $A=52^{\circ}16'48.72''$	$R=3795.486$ $B=40^{\circ}36'10.9''$ $C=87^{\circ}7'0.38''$ $c=7581.363$	$r=1599.1825$ $r_1=4545.994$ $r_2=3426.808$ $r_3=8808.323$
30.	$a=72027.91$ $b=63926$ $A=76^{\circ}22'16.08''$	$B=37^{\circ}57.4$ $B=59^{\circ}36'5.30''$ $C=44^{\circ}1'38.62''$ $c=51509.95$	$r=17070.53$ $r_1=73721.52$ $r_2=53682.39$ $r_3=37896.22$
31.	$a=10301.99$ $b=9205.4$ $A=61^{\circ}16'11.5''$	$R=5874.135$ $B=51^{\circ}35'13.32''$ $C=67^{\circ}8'35.18''$ $c=10825.782$	$r=2880.918$ $r_1=8981.956$ $r_2=7329.696$ $r_3=10065.804$
32.	$a=72027.91$ $b=58443.05$ $A=86^{\circ}18'43.14''$	$R=36088.7$ $B=54^{\circ}4'4.86''$ $C=39^{\circ}37'12''$ $c=46027$	$r=15209.16$ $r_1=82743.85$ $r_2=45031.10$ $r_3=31788.98$
33.	$a=12785.2$ $b=11688.61$ $A=69^{\circ}4'52.18''$	$R=6843.687$ $B=58^{\circ}38'46.20''$ $C=52^{\circ}16'21.62''$ $c=10825.782$	$r=3348.419$ $r_1=12148.782$ $r_2=9913.952$ $r_3=8660.432$

IX.—Triangles in which two sides and the angle opposite the lesser are given. Ambiguous Case.

Let the two given lengths be a, b ($a > b$), and B the given angle. First calculate R by the equation $2R = b/\sin B$; then $\sin A$ may be found from $\sin A = a/2R$, but either the acute or the obtuse angle having this sine may be taken, and there are two triangles having the assigned data in all cases except when $A = 90^{\circ}$. Thus there will be two values of A , supplementary to each other. We shall denote the acute angle by A_1 , the obtuse by A_2 . The equation $A + B + C = 180^{\circ}$ supplies two corresponding values of C , which we denote by C_1, C_2 . The work is otherwise exactly the same as in VIII. The equation $b^2 = a^2 + c^2 - 2ac \cos B$ shows us that, if c_1, c_2 be the two values of c , $2a + c_1 = c_2 \cos B$, $c_1 c_2 = a^2 - b^2 \equiv (a - b)(a + b)$, and either of these may be used

Calculation of the Dihedral Angles.

$-L \sin \sigma_1 = -9.8812306$ $L \sin (\sigma_1 - \alpha_1) = 9.9004600$ $L \sin (\sigma_1 - \beta_1) = 9.8805092$ $L \sin (\sigma_1 - \gamma_1) = 9.6770079$ 2) 19.5767525	19.7883763 9.9004660 $L \tan \frac{1}{2} X$ 9.8879103	19.7883763 9.8805092 $L \tan \frac{1}{2} Y$ 9.9078671	19.7883763 9.6770079 $L \tan \frac{1}{2} Z$ 10.1113684	-9.9847348 9.3307318 9.7612771 9.5300101 2) 18.8372842	5 3 4 4 6	19.4186421 9.5307318 $L \tan \frac{1}{2} X$ 9.8879103	19.4186421 9.7612771 $L \tan \frac{1}{2} B$ 9.6573650	19.4186421 9.5300101 $L \tan \frac{1}{2} C$ 9.8886320
$-L \sin \sigma_2 = -9.9860869$ $L \sin (\sigma_2 - \alpha_2) = 9.7825859$ $L \sin (\sigma_2 - \beta_2) = 9.5037795$ $L \sin (\sigma_2 - \gamma_2) = 9.5250146$ 2) 18.8272932	19.4136466 9.7825860 $L \tan \frac{1}{2} A$ 9.6310606	19.4136466 9.5057796 $L \tan \frac{1}{2} Y$ 9.9078670	19.4136466 9.5250146 $L \tan \frac{1}{2} C$ 9.8886320	-9.9920300 9.7607634 9.7344588 9.2804556 18.7836478	5 4 1 5 5	19.3918239 9.7607634 $L \tan \frac{1}{2} A$ 9.6310605	19.3918239 9.7344588 $L \tan \frac{1}{2} B$ 9.6573651	19.3918239 9.2804556 $L \tan \frac{1}{2} Z$ 10.1113683
$L \tan \frac{1}{2} A = 9.6310606$ $23^\circ 9' + \left\{ \begin{array}{l} 052 \\ 1803 \\ 56 \end{array} \right\}$ $3493 \left\{ \begin{array}{l} 554 \\ 8554 \\ 1608 \end{array} \right\}$ $L \tan \frac{1}{2} X = 9.8879103$ $37^\circ 41' + \left\{ \begin{array}{l} 2611 \end{array} \right\}$	$33240(9.52$ 1803 56 554 $32940(12.62$ 6830 1608 41	$L \tan \frac{1}{2} B = 9.6573651$ $24^\circ 26' + \left\{ \begin{array}{l} 36 \\ 3353 \end{array} \right\}$ $L \tan \frac{1}{2} Y = 9.9078671$ $38^\circ 58' + \left\{ \begin{array}{l} 525 \\ 2584 \end{array} \right\}$	$L \tan \frac{1}{2} C = 9.8886320$ $37^\circ 44' - \left\{ \begin{array}{l} 2611 \end{array} \right\}$ $L \tan \frac{1}{2} Z = 10.1113684$ $52^\circ 16' + \left\{ \begin{array}{l} 2611 \end{array} \right\}$	$900(.27$ 229 $8760(3.39$ 1008 233	$L \tan \frac{1}{2} C = 9.8886320$ 36 66 14 70	$3960(1.52$ 1349 43 $4200(1.61$ 1589		

ANSWERS.

$\frac{1}{2} A = 23^\circ 0' 9.50''$ $\frac{1}{2} X = 37^\circ 41' 12.63''$	$\frac{1}{2} B = 24^\circ 26' 0.25''$ $\frac{1}{2} Y = 38^\circ 58' 3.38''$	$\frac{1}{2} C = 37^\circ 43' 58.48''$ $\frac{1}{2} Z = 52^\circ 16' 1.61''$	$V_1 = 8.63133$ $V_2 = 8.631332$ $V_3 = 8.631332$ $V_4 = 8.63133$	$\sigma_1 = 130^\circ 28' 17.03''$ $\sigma_2 = 74^\circ 53' 50.95''$ $\sigma_3 = 75^\circ 34' 27.67''$ $\sigma_4 = 79^\circ 3' 24.35''$
$\frac{1}{2} (A + X) = 60^\circ 50' 22.13''$ $\frac{1}{2} (X - A) = 14^\circ 32' 3.13''$ $a + x = 8.5475$ $x - a = 1.4109$	$\frac{1}{2} (B + Y) = 63^\circ 24' 3.63''$ $\frac{1}{2} (Y - B) = 14^\circ 32' 3.13''$ $b + y = 8.7643$ $y - b = 1.4109$	$\frac{1}{2} (Z + C) = 90^\circ 0' 0.09''$ $\frac{1}{2} (Z - C) = 14^\circ 32' 3.13''$ $z + c = 9.8579$ $z - c = 1.4109$		

EXAMPLES. X.

	DATA.	$\frac{1}{2}a_0, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_0, \frac{1}{2}b_2, \frac{1}{2}c_1$	$\frac{1}{2}a_0, \frac{1}{2}b_3, \frac{1}{2}c_1$	$\frac{1}{2}a_0, \frac{1}{2}b_4, \frac{1}{2}c_1$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
1.	$\begin{cases} a=5.4669, \\ b=5.6185, \\ c=5.6601, \end{cases}$ $\begin{cases} x=7.7454, \\ y=8.1712, \\ z=8.1836 \end{cases}$	$\begin{cases} 43^\circ 37' 59.08'' \\ 47^\circ 35' 7.24'' \\ 47^\circ 34' 3.13'' \end{cases}$	$\begin{cases} 28^\circ 16' 2.20'' \\ 21^\circ 34' 11.80'' \\ 21^\circ 33' 12.17'' \end{cases}$	$\begin{cases} 20^\circ 59' 33.47'' \\ 30^\circ 49' 33.47'' \\ 23^\circ 9' 0.95'' \end{cases}$	$\begin{cases} 20^\circ 53' 40.59'' \\ 23^\circ 12' 59.97'' \\ 30^\circ 54' 24.33'' \end{cases}$	$\begin{cases} 24^\circ 26' 10.46'' \\ 27^\circ 26' 34.59'' \\ 27^\circ 28' 3.86'' \end{cases}$	$\begin{cases} 43^\circ 51' 10.86'' \\ 47^\circ 26' 29.40'' \\ 47^\circ 28' 0.18'' \end{cases}$	28.44716	$S_1 = \sigma_1 = 138^\circ 45' 40.44''$
2.	$\begin{cases} a=7.4779, \\ b=8.1235, \\ c=8.4567, \end{cases}$ $\begin{cases} x=5.4778, \\ y=4.8322, \\ z=4.4990 \end{cases}$	$\begin{cases} 19^\circ 15' 31.94'' \\ 17^\circ 18' 32.99'' \\ 16^\circ 35' 47.81'' \end{cases}$	$\begin{cases} 35^\circ 53' 48.79'' \\ 40^\circ 39' 13.97'' \\ 41^\circ 55' 14.18'' \end{cases}$	$\begin{cases} 32^\circ 44' 58.22'' \\ 28^\circ 27' 49.45'' \\ 37^\circ 0' 59.50'' \end{cases}$	$\begin{cases} 30^\circ 46' 12.83'' \\ 33^\circ 43' 28.56'' \\ 25^\circ 38' 21.76'' \end{cases}$	$\begin{cases} 35^\circ 24' 48.19'' \\ 41^\circ 25' 24.17'' \\ 43^\circ 9' 58.46'' \end{cases}$	$\begin{cases} 36^\circ 13' 44.26'' \\ 30^\circ 13' 8.23'' \\ 28^\circ 28' 34.01'' \end{cases}$	25.39085	
3.	$\begin{cases} a=7.4779, \\ b=8.1235, \\ c=8.4567, \end{cases}$ $\begin{cases} x=5.4778, \\ y=4.8322, \\ z=4.4990 \end{cases}$	$\begin{cases} 18^\circ 1' 21.39'' \\ 15^\circ 41' 40.71'' \\ 16^\circ 35' 47.81'' \end{cases}$	$\begin{cases} 35^\circ 53' 48.79'' \\ 40^\circ 39' 13.97'' \\ 47^\circ 27' 2.80'' \end{cases}$	$\begin{cases} 32^\circ 44' 58.22'' \\ 28^\circ 27' 49.45'' \\ 41^\circ 55' 14.18'' \end{cases}$	$\begin{cases} 26^\circ 51' 16.49'' \\ 30^\circ 22' 56.06'' \\ 25^\circ 38' 21.76'' \end{cases}$	$\begin{cases} 32^\circ 33' 45.21'' \\ 39^\circ 35' 7.60'' \\ 49^\circ 3' 38.80'' \end{cases}$	$\begin{cases} 35^\circ 21' 19.15'' \\ 28^\circ 19' 56.74'' \\ 30^\circ 42' 38.45'' \end{cases}$	25.18782	$S_2 = \sigma_2 = 124^\circ 0' 5.55''$
4.	$\begin{cases} a=3.3333, \\ b=3.1111, \\ c=4.668152, \end{cases}$ $\begin{cases} x=4.1114, \\ y=3.0002, \\ z=3.2224 \end{cases}$	$\begin{cases} 29^\circ 56' 59.97'' \\ 19^\circ 54' 42.40'' \\ 29^\circ 56' 34.68'' \end{cases}$	$\begin{cases} 41^\circ 18' 47.86'' \\ 28^\circ 18' 56.73'' \\ 47^\circ 24' 30.53'' \end{cases}$	$\begin{cases} 31^\circ 44' 28.59'' \\ 23^\circ 10' 48.39'' \\ 39^\circ 56' 10.95'' \end{cases}$	$\begin{cases} 22^\circ 40' 47.07'' \\ 20^\circ 26' 49.08'' \\ 25^\circ 30' 23.75'' \end{cases}$	$\begin{cases} 31^\circ 44' 53.38'' \\ 27^\circ 42' 55.74'' \\ 50^\circ 22' 39.20'' \end{cases}$	$\begin{cases} 38^\circ 56' 50.18'' \\ 23^\circ 11' 2.35'' \\ 38^\circ 55' 41.93'' \end{cases}$	4.370334	$S_3 = \sigma_3 = 117^\circ 2' 15.12''$
5.	$\begin{cases} a=5.2669, \\ b=7.8712, \\ c=7.8836, \end{cases}$ $\begin{cases} x=7.47234, \\ y=5.4185, \\ z=5.4061 \end{cases}$	$\begin{cases} 28^\circ 18' 47.07'' \\ 21^\circ 36' 9.84'' \\ 21^\circ 35' 7.92'' \end{cases}$	$\begin{cases} 43^\circ 39' 16.88'' \\ 47^\circ 39' 47.98'' \\ 47^\circ 32' 18.12'' \end{cases}$	$\begin{cases} 20^\circ 54' 4.10'' \\ 23^\circ 12' 25.39'' \\ 30^\circ 53' 7.14'' \end{cases}$	$\begin{cases} 20^\circ 51' 32.04'' \\ 30^\circ 48' 5.79'' \\ 23^\circ 8' 17.73'' \end{cases}$	$\begin{cases} 24^\circ 27' 40.25'' \\ 47^\circ 24' 21.59'' \\ 47^\circ 25' 55.78'' \end{cases}$	$\begin{cases} 43^\circ 52' 5.60'' \\ 27^\circ 30' 1.09'' \\ 27^\circ 28' 27.88'' \end{cases}$	25.49455	$S_4 = \sigma_4 = 138^\circ 42' 22.97''$
6.	$\begin{cases} a=2.6335, \\ b=2.7093, \\ c=2.7031, \end{cases}$ $\begin{cases} x=3.7942, \\ y=3.8356, \\ z=3.8418 \end{cases}$	$\begin{cases} 44^\circ 30' 31.86'' \\ 45^\circ 56' 43.46'' \\ 45^\circ 58' 17.20'' \end{cases}$	$\begin{cases} 29^\circ 37' 2.01'' \\ 22^\circ 24' 25.69'' \\ 22^\circ 23' 19.59'' \end{cases}$	$\begin{cases} 21^\circ 37' 17.11'' \\ 30^\circ 9' 2.48'' \\ 22^\circ 42' 43.89'' \end{cases}$	$\begin{cases} 21^\circ 39' 56.95'' \\ 22^\circ 19' 54.39'' \\ 30^\circ 13' 55.51'' \end{cases}$	$\begin{cases} 26^\circ 0' 44.56'' \\ 27^\circ 32' 57.96'' \\ 27^\circ 31' 22.62'' \end{cases}$	$\begin{cases} 44^\circ 32' 25.39'' \\ 45^\circ 55' 45.88'' \\ 45^\circ 57' 21.25'' \end{cases}$	3.210399	$S_5 = \sigma_5 = 136^\circ 25' 32.52''$
7.	$\begin{cases} a=2.6155, \\ b=2.6093, \\ c=2.6031, \end{cases}$ $\begin{cases} x=3.7116, \\ y=3.6356, \\ z=3.6418 \end{cases}$	$\begin{cases} 45^\circ 24' 13.12'' \\ 44^\circ 9' 34.68'' \\ 44^\circ 11' 18.65'' \end{cases}$	$\begin{cases} 30^\circ 39' 53.80'' \\ 22^\circ 52' 14.79'' \\ 22^\circ 51' 0.31'' \end{cases}$	$\begin{cases} 22^\circ 56' 26.56'' \\ 29^\circ 37' 34.91'' \\ 22^\circ 15' 52.49'' \end{cases}$	$\begin{cases} 22^\circ 59' 25.01'' \\ 22^\circ 19' 54.39'' \\ 30^\circ 13' 55.51'' \end{cases}$	$\begin{cases} 28^\circ 18' 19.42'' \\ 27^\circ 21' 8.69'' \\ 27^\circ 19' 26.23'' \end{cases}$	$\begin{cases} 45^\circ 25' 40.05'' \\ 44^\circ 8' 51.98'' \\ 44^\circ 10' 34.47'' \end{cases}$	2.958072	$S_6 = \sigma_6 = 133^\circ 45' 6.48''$
8.	$\begin{cases} a=4.4877, \\ b=5.4877, \\ c=6.6877, \end{cases}$ $\begin{cases} x=6.6466, \\ y=4.9493, \\ z=4.4466 \end{cases}$	$\begin{cases} 32^\circ 39' 13.33'' \\ 23^\circ 52' 5.53'' \\ 25^\circ 52' 59.29'' \end{cases}$	$\begin{cases} 44^\circ 56' 27.89'' \\ 37^\circ 53' 42.32'' \\ 45^\circ 3' 29.68'' \end{cases}$	$\begin{cases} 26^\circ 13' 18.39'' \\ 24^\circ 3' 49.92'' \\ 33^\circ 2' 41.31'' \end{cases}$	$\begin{cases} 21^\circ 4' 24.79'' \\ 24^\circ 18' 5.36'' \\ 20^\circ 59' 42.19'' \end{cases}$	$\begin{cases} 30^\circ 3' 56.67'' \\ 37^\circ 53' 41.77'' \\ 45^\circ 4' 30.01'' \end{cases}$	$\begin{cases} 44^\circ 55' 28.11'' \\ 27^\circ 16' 14.54'' \\ 29^\circ 54' 54.77'' \end{cases}$	15.95682	$S_7 = \sigma_7 = 127^\circ 53' 39.89''$
9.	$\begin{cases} a=4.6739, \\ b=5.5587, \\ c=4.4435, \end{cases}$ $\begin{cases} x=6.6572, \\ y=6.5586, \\ z=6.4434 \end{cases}$	$\begin{cases} 47^\circ 41' 6.93'' \\ 45^\circ 58' 59.34'' \\ 44^\circ 15' 14.06'' \end{cases}$	$\begin{cases} 30^\circ 47' 39.37'' \\ 22^\circ 30' 22.25'' \\ 21^\circ 18' 32.58'' \end{cases}$	$\begin{cases} 23^\circ 14' 23.69'' \\ 30^\circ 1' 43.41'' \\ 20^\circ 49' 25.06'' \end{cases}$	$\begin{cases} 22^\circ 42' 28.08'' \\ 21^\circ 29' 28.01'' \\ 29^\circ 10' 37.22'' \end{cases}$	$\begin{cases} 28^\circ 22' 50.84'' \\ 26^\circ 35' 44.75'' \\ 25^\circ 1' 14.87'' \end{cases}$	$\begin{cases} 47^\circ 39' 43.28'' \\ 45^\circ 55' 3.45'' \\ 44^\circ 20' 33.60'' \end{cases}$	15.69689	$S_8 = \sigma_8 = 137^\circ 55' 20.33''$
10.	$\begin{cases} a=8.7737, \\ b=7.8837, \\ c=7.7745, \end{cases}$ $\begin{cases} x=7.8848, \\ y=8.7748, \\ z=8.6656 \end{cases}$	$\begin{cases} 30^\circ 13' 59.65'' \\ 31^\circ 51' 17.90'' \\ 31^\circ 12' 45.19'' \end{cases}$	$\begin{cases} 26^\circ 52' 34.55'' \\ 26^\circ 52' 26.72'' \\ 26^\circ 17' 50.37'' \end{cases}$	$\begin{cases} 31^\circ 54' 48.09'' \\ 31^\circ 54' 56.80'' \\ 29^\circ 32' 26.10'' \end{cases}$	$\begin{cases} 31^\circ 50' 51.73'' \\ 30^\circ 13' 34.55'' \\ 31^\circ 12' 28.05'' \end{cases}$	$\begin{cases} 36^\circ 54' 45.66'' \\ 34^\circ 22' 33.72'' \\ 33^\circ 19' 22.88'' \end{cases}$	$\begin{cases} 34^\circ 22' 48.48'' \\ 36^\circ 55' 0.43'' \\ 35^\circ 51' 49.59'' \end{cases}$	66.38926	$a + x = b + y, y - b = z - c$

to check the work. Since the angle B is common to both triangles, this angle should be used in calculating r , r_1 , r_2 , r_3 for the two triangles.

EXAMPLE WORKED OUT.

DATA.	$\log b=4.1673297$ $-\log \sin B=1.9375306$ $\log 2R=4.2297991$ $\frac{842}{149}$ 16974.58	$\log a=4.2273377$ $-\log 2R=4.2297991$ $L \sin A=9.9975386$ $83^\circ 54' + \}$ 135	$2760(20.44''$ $A_1=83^\circ 54' 20.44''$ $C_1=36^\circ 5' 39.56''$ $A_2=96^\circ 5' 39.56''$ $C_2=23^\circ 54' 20.44''$
$a=16878.65$ $b=14700.42$ $B=60^\circ$			
	$\log 2R=4.2297991$ $\log \sin C_1=1.7700868$ $\frac{1143}{}$ $\log c_1=4.0000002$ <u>10000.005</u>	$\frac{1733}{}$ $\frac{36=1039}{3.6=104}$ $-.04= -1$ 2	$\log 2R=4.2297991$ $-\log \sin C_2=1.6076068$ $\frac{971}{}$ $\log c_2=3.8375030$ $\frac{01}{}$ <u>6878.645</u> $\log(a-b)=3.3381038$ $\log(a+b)=4.4993994$ $\log(a^2-b^2)=7.8375032$ $\log c_1 c_2=7.8375032$
			2850 $20=950$ $.4=19$ $.04=2$
			$\log \tan \frac{B}{2}=1.7614394$ $\log \cot \frac{1}{2}B=0.2385606$

$a=16878.65$ $b=14700.42$ $c=10000.01$ $2s_1=41579.08$ $s_1=20789.54$	$s_1-a=3910.89$ $s_1-b=6089.12$ $s_1-c=10789.53$ $s_1=20789.54$	$\log=3.5922756$ $\log=3.7845545$ $\log=4.0330024$ $\log=4.3178449$	
I. $\log r$	$\log r_1$	$\log r_2$	$\log r_3$
3.7845545 1.7614394	4.0330024 $.2385606$	4.3178449 1.7614394	3.5922756 $.2385606$
3.5459939 871	4.2715630 28	4.0792843 536	3.8308362 24
<u>3515.555</u> 68	<u>18688.01</u> 2	<u>12002.85</u> 307	<u>6773.86</u> 38
			$-r=3515.555$ $r_1=18688.01$ $r_2=12002.85$ $r_3=6773.86$ <u>$4R=33949.165$</u>

$a=16878.65$ $b=14700.42$ $c_2=6878.65$ $2s_2=38457.72$ $s_2=19228.86$	$s_2-a=2350.21$ $s_2-b=4528.44$ $s_2-c_2=12350.21$ $s_2=19228.86$	$\log=3.3711067$ $\log=3.6559486$ $\log=4.0916743$ $\log=4.2839535$	
II. $\log r$	$\log r_1$	$\log r_2$	$\log r_3$
3.6559486 1.7614394	4.0916743 $.2385606$	4.2839535 1.7614394	3.3711067 $.2385606$
3.4173880 720	4.3302349 11	4.0453929 621	3.6096673 584
<u>2614.497</u> 160	<u>21391.19</u> 38	<u>11101.79</u> 308	<u>4070.683</u> 89
			$-r=2614.497$ $r_1=21391.19$ $r_2=11101.79$ $r_3=4070.683$ <u>$4R=33949.166$</u>

ANSWERS (I.).

$R=8487.29$
 $A=83^\circ 54' 20.44''$
 $C=36^\circ 5' 39.56''$
 $c=10000.005$

$r=3515.555$
 $r_1=18688.01$
 $r_2=12002.85$
 $r_3=6773.86$

ANSWERS (II.).

$R=8487.29$
 $A=96^\circ 5' 39.56''$
 $C=23^\circ 54' 20.44''$
 $c=6878.645$

$r=2614.497$
 $r_1=21391.19$
 $r_2=11101.79$
 $r_3=4070.683$

EXAMPLES IX.

DATA.

ANSWERS (1).

ANSWERS (2).

1. $a=38^{\circ}30.683$ $b=33^{\circ}06.549$ $B=62^{\circ}27'27.4''$		$R=1915.342$ $A=90^{\circ}$ $C=27^{\circ}32'32.6''$ $c=1771.326$	$r=668.596$ $r_1=4499.279$ $r_2=2727.953$ $r_3=1102.73$	Same as (1)	Same as (1)
2. $a=270.5115$ $b=116.5868$ $B=25^{\circ}31'43.6''$	$\Delta_1=78.09004$	$R=135.2558$ $A=90^{\circ}$ $C=64^{\circ}28'16.4''$ $c=244.1011$	$r=45.0852$ $r_1=315.5967$ $r_2=71.4956$ $r_3=199.0159$	Same as (1)	Same as (1)
3. $a=3650.401$ $b=2107.56$ $B=30^{\circ}$		$R=2107.56$ $A=60^{\circ}$ $C=90^{\circ}$ $c=4215.12$	$r=771.4205$ $r_1=2878.9805$ $r_2=1336.1395$ $r_3=4986.5405$	$R=2107.56$ $A=120^{\circ}$ $C=30^{\circ}$ $c=2107.56$	$r=489.0611$ $r_1=6811.7405$ $r_2=1053.78$ $r_3=1053.78$
4. $a=24150.16$ $b=22202.21$ $B=50^{\circ}59'37.5''$	(1) $\Delta_1=2534.15$ (2) $\Delta_1=18006.17$	$R=14285.71$ $A=57^{\circ}41'57.5''$ $C=71^{\circ}18'25''$ $c=27064.26$	$r=6918.088$ $r_1=20222.02$ $r_2=17506.51$ $r_3=20332.42$	$R=14285.71$ $A=122^{\circ}18'2.5''$ $C=6^{\circ}42'20''$ $c=3336.198$	$r=1260.025$ $r_1=45999.07$ $r_2=11848.4$ $r_3=1455.472$
5. $a=10316.36$ $b=6685.65$ $B=39^{\circ}53'33''$	(1) $\Delta_1=1873.9565$ (2) $\Delta_1=2672.551$	$R=5212.165$ $A=81^{\circ}44'47.09''$ $C=58^{\circ}21'39.91''$ $c=8874.97$	$r=2269.206$ $r_1=11107.104$ $r_2=4695.632$ $r_3=7225.329$	$R=5212.165$ $A=98^{\circ}15'12.91''$ $C=41^{\circ}51'14.09''$ $c=6955.463$	$r=1020.904$ $r_1=13841.724$ $r_2=4347.183$ $r_3=4580.709$
6. $a=10316.36$ $b=6683.635$ $B=39^{\circ}53'33''$	(1) $\Delta_1=1877.5675$ (2) $\Delta_1=2664.942$	$R=5210.6665$ $A=81^{\circ}51'55.83''$ $C=58^{\circ}14'31.17''$ $c=8860.933$	$r=2267.026$ $r_1=11213.669$ $r_2=4692.57$ $r_3=7203.212$	$R=5210.6665$ $A=98^{\circ}8'4.17''$ $C=41^{\circ}58'22.83''$ $c=6969.503$	$r=1023.817$ $r_1=13819.64$ $r_2=4349.364$ $r_3=4597.275$
7. $a=346.4537$ $b=301.3984$ $B=60^{\circ}153.4''$		$R=173.95725$ $A=84^{\circ}44'52.3''$ $C=35^{\circ}13'14.3''$ $c=200.6515$	$r=70.97449$ $r_1=387.0413$ $r_2=245.09745$ $r_3=134.6647$	$R=173.95725$ $A=95^{\circ}15'7.7''$ $C=24^{\circ}42'58.9''$ $c=145.3837$	$r=55.00991$ $r_1=434.8742$ $r_2=229.1328$ $r_3=86.8318$
8. $a=3156.045$ $b=3013.734$ $B=72^{\circ}13'57.6''$		$R=1582.339$ $A=85^{\circ}46'1.3''$ $C=22^{\circ}0'1.1''$ $c=1185.525$	$r=484.4273$ $r_1=3415.513$ $r_2=2683.396$ $r_3=714.8735$	$R=1582.339$ $A=94^{\circ}13'58.7''$ $C=13^{\circ}32'3.7''$ $c=740.625$	$r=322.116$ $r_1=3720.385$ $r_2=2521.085$ $r_3=410.002$

EXAMPLES IX. — Continued.

DATA.	ANSWERS (1).			ANSWERS (2).		
9. $a=7631.229$ $b=4973.381$ $B=39^{\circ}53'46.45''$	$R=3876.978$ $A=79^{\circ}47'32.86''$ $C=60^{\circ}18'40.69''$ $c=6736.086$	$r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$	$R=3876.978$ $A=100^{\circ}12'27.14''$ $C=39^{\circ}53'46.41''$ $c=4973.38$	$r=1384.858$ $r_1=10512.927$ $r_2=3189.923$ $r_3=3189.920$		
10. $a=4731.234$ $b=2718.108$ $B=32^{\circ}3'57.6''$	$R=2560.001$ $A=67^{\circ}31'40.31''$ $C=80^{\circ}24'22.09''$ $c=5048.411$	$r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$	$R=2560.011$ $A=112^{\circ}28'19.69''$ $C=35^{\circ}27'42.71''$ $c=2970.436$	$r=716.047$ $r_1=7793.115$ $r_2=1497.173$ $r_3=1665.803$		
11. $a=481.2793$ $b=360.8528$ $B=46^{\circ}44'10.4''$	$R=247.7683$ $A=76^{\circ}13'21.66''$ $C=57^{\circ}27.94''$ $c=415.7854$	$r=115.8427$ $r_1=493.3678$ $r_2=271.7591$ $r_3=341.7890$	$R=247.7683$ $A=103^{\circ}46'38.34''$ $C=29^{\circ}29'11.26''$ $c=243.9119$	$r=78.7113$ $r_1=692.2594$ $r_2=234.6279$ $r_3=142.8972$		
12. $a=59406.24$ $b=22733.78$ $B=22^{\circ}30'$	$R=29703.12$ $A=90^{\circ}$ $C=67^{\circ}30'$ $c=54884.2$	$r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$	Same as (1)	Same as (1)		
13. $a=964701$ $b=955078$ $B=59^{\circ}59'23.7''$	(1) $\Delta_1=9628.197$ (2) $\Delta_1=542988.5$	$R=551470.5$ $A=61^{\circ}0'18.17''$ $C=59^{\circ}0'18.13''$ $c=9454.9$	$R=551470.5$ $A=118^{\circ}59'41.83''$ $C=1^{\circ}0'54.47''$ $c=19539.84$	$r=8416.88$ $r_1=1645989.8$ $r_2=559719.5$ $r_3=8589.98$		
14. $a=337.573$ $b=294.0084$ $B=60^{\circ}$	(1) $\Delta_1=70.311$ $\Delta_2=70.311$ (2) $\Delta_1=105.17894$	$R=169.74584$ $A=83^{\circ}54'20.44''$ $C=36^{\circ}5'39.56''$ $c=200$	$R=169.74584$ $A=96^{\circ}5'39.56''$ $C=23^{\circ}54'20.44''$ $c=137.57292$	$r=52.28994$ $r_1=427.82382$ $r_2=222.03568$ $r_3=81.41366$		
15. $a=6142.887$ $b=6127.56$ $B=59^{\circ}45'12''$	(1) $\Delta_1=30.539$ (2) $\Delta_2=6158.099$	$R=3546.597$ $A=60^{\circ}$ $C=60^{\circ}14'18''$ $c=6158.099$	$R=3546.597$ $A=120^{\circ}$ $C=0^{\circ}14'48''$ $c=30.539$	$r=13.17464$ $r_1=10652.965$ $r_2=3533.357$ $r_3=13.23971$		
16. $a=775.2704$ $b=674.2277$ $B=60^{\circ}$	(1) $\Delta_1=\Delta_2=166.874$ $\Delta_3=325.9911$ (2) $\Delta_1=\Delta_2=235.7022$ $\Delta_3=449.2793$	$R=389.2655$ $A=84^{\circ}45'15''$ $C=35^{\circ}14'45''$ $c=449.2793$	$R=389.2655$ $A=95^{\circ}14'45''$ $C=24^{\circ}45'15''$ $c=325.9911$	$r=123.2741$ $r_1=372.9854$ $r_2=512.5396$ $r_3=194.8110$		

DATA.			ANSWERS (1).			ANSWERS (2).		
17.	$a=701.37$ $b=666.3$ $B=54^{\circ} 9.25''$	$R=411.7826$ $A=58^{\circ} 23' 20.5''$ $C=67^{\circ} 36' 30.25''$ $c=761.47$	$r=202.9399$ $r_1=594.8341$ $r_2=542.4556$ $r_3=712.7807$	$R=411.7826$ $A=121^{\circ} 36' 39.5''$ $C=4^{\circ} 23' 11.25''$ $c=629.89$	$r=24.9832$ $r_1=1280.219$ $r_2=364.4987$ $r_3=27.3955$			
18.	$a=393.75$ $b=391.54$ $B=64^{\circ} 46' 15.36''$	$R=216.4135$ $A=65^{\circ} 27' 58.58''$ $C=49^{\circ} 45' 46.06''$ $c=330.41$	$r=105.4844$ $r_1=358.589$ $r_2=353.8239$ $r_3=258.7252$	$R=216.4135$ $A=114^{\circ} 32' 1.42''$ $C=0^{\circ} 41' 43.22''$ $c=5.25254$	$r=2.36661$ $r_1=614.9157$ $r_2=250.7062$ $r_3=2.39848$			
19.	$a=5265.68$ $b=4822.3$ $B=65^{\circ} 28' 2.86''$	$R=2650.415$ $A=83^{\circ} 23' 53.3''$ $C=31^{\circ} 8' 3.84''$ $c=2740.78$	$r=1023.421$ $r_1=5714.817$ $r_2=4123.293$ $r_3=1786.97$	$R=2650.415$ $A=96^{\circ} 36' 6.7''$ $C=17^{\circ} 55' 50.44''$ $c=1631.947$	$r=667.0308$ $r_1=6577.292$ $r_2=3766.902$ $r_3=924.4924$			
20.	$a=688.9$ $b=627.77$ $B=64^{\circ} 16' 8.94''$	$R=348.4346$ $A=81^{\circ} 19' 36.02''$ $C=34^{\circ} 24' 15.04''$ $c=393.75$	$r=142.8642$ $r_1=734.6451$ $r_2=537.1917$ $r_3=264.7658$	$R=348.4346$ $A=98^{\circ} 40' 23.98''$ $C=17^{\circ} 32' 27.08''$ $c=204.414$	$r=83.40002$ $r_1=885.3568$ $r_2=477.7268$ $r_3=114.0542$			
21.	$a=313.885$ $b=165.205$ $B=25^{\circ} 44' 5.06''$	$R=190.2383$ $A=55^{\circ} 35' 10.62''$ $C=68^{\circ} 40' 44.32''$ $c=376.12$	$r=59.94535$ $r_1=225.3852$ $r_2=97.67835$ $r_3=497.8303$	$R=190.2383$ $A=124^{\circ} 24' 49.38''$ $C=29^{\circ} 51' 5.56''$ $c=189.384$	$r=38.61218$ $r_1=634.1209$ $r_2=76.35015$ $r_3=89.09454$			
22.	$a=10978.38$ $b=9662.8$ $B=54^{\circ} 43' 38.54''$	$R=5917.834$ $A=68^{\circ} 31.5''$ $C=57^{\circ} 12' 49.96''$ $c=9950.22$	$r=2915.29$ $r_1=10328.493$ $r_2=7916.244$ $r_3=8341.887$	$R=5917.834$ $A=111^{\circ} 56' 28.5''$ $C=13^{\circ} 19' 52.96''$ $c=2729.1$	$r=1046.656$ $r_1=17304.79$ $r_2=6047.607$ $r_3=1365.596$			
23.	$a=1998.237$ $b=633.911$ $B=39^{\circ} 20' 21.35''$	$R=500$ $A=86^{\circ} 35' 50.4''$ $C=3^{\circ} 48.25''$ $c=809.6669$	$r=209.8239$ $r_1=1150.473$ $r_2=436.4176$ $r_3=622.9357$	$R=500$ $A=93^{\circ} 24' 9.6''$ $C=47^{\circ} 15' 29.05''$ $c=734.4185$	$r=196.375$ $r_1=1255.73$ $r_2=422.9687$ $r_3=517.6794$			
24.	$a=4457.15$ $b=4164.894$ $B=54^{\circ} 1' 18.24''$	$R=2573.337$ $A=60^{\circ}$ $C=65^{\circ} 58' 41.76''$ $c=4700.928$	$r=1272.674$ $r_1=3846.01$ $r_2=3395.787$ $r_3=4324.224$	$R=2573.337$ $A=120^{\circ}$ $C=5^{\circ} 58' 41.76''$ $c=536.0318$	$r=211.116$ $r_1=7931.125$ $r_2=2334.23$ $r_3=239.1052$			

X.—Tetrahedra, the lengths of the edges being given.

In a tetrahedron $OABC$ let the lengths of OA, OB, OC be denoted by a, b, c ; the lengths of the respectively opposite edges BC, CA, AB by x, y, z ; the dihedral angles opposite to a, b, c, x, y, z respectively by A, B, C, X, Y, Z . Also, let any angle of a triangular face which is opposite to OA or BC be denoted by α , one opposite to OB or CA by β , one opposite to OC or AB by γ ; all the angles at O having the suffix 1, those at A the suffix 2, those at B the suffix 3, and those at C the suffix 4. Then, in each of the four triangular faces we have the lengths of the three sides given, and may calculate the angles as in VI. In the face ABC opposite O the lengths are x, y, z , the angles $\alpha_2, \beta_3, \gamma_4$; in the face opposite A the lengths are x, b, c , the angles $\alpha_1, \beta_4, \gamma_3$; in the face opposite B the lengths are a, y, c , the angles $\alpha_4, \beta_1, \gamma_2$; lastly, in the face opposite C the lengths are a, b, z , the angles $\alpha_3, \beta_2, \gamma_1$. The sums of the lengths are denoted by $2s_1, 2s_2, 2s_3, 2s_4$. The sum of the plane angles at O is similarly $2\sigma_1$, and the sums of the angles at A, B, C are $2\sigma_2, 2\sigma_3, 2\sigma_4$. The volume V may be found from each of the equations

$$\begin{aligned} 3V &= abc \sqrt{\sin \sigma_1 \sin (\sigma_1 - \alpha_1) \sin (\sigma_1 - \beta_1) \sin (\sigma_1 - \gamma_1)} \\ &= ayz \sqrt{\sin \sigma_2 \sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2)} \\ &= xbz \sqrt{\sin \sigma_3 \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3)} \\ &= xyc \sqrt{\sin \sigma_4 \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4)}. \end{aligned}$$

The most convenient manner of arranging the work will be obvious on inspecting the one calculated. The dihedral angles at O are X, Y, Z , and these are to be determined by the equations

$$\begin{aligned} \tan \frac{1}{2} X \sin (\sigma_1 - \alpha_1) &= \tan \frac{1}{2} Y \sin (\sigma_1 - \beta_1) = \tan \frac{1}{2} Z \sin (\sigma_1 - \gamma_1) \\ &= \sqrt{\frac{\sin (\sigma_1 - \alpha_1) \sin (\sigma_1 - \beta_1) \sin (\sigma_1 - \gamma_1)}{\sin \sigma_1}}, \end{aligned}$$

no fresh logarithms needing to be taken out. This part of the work is almost exactly the same as that of calculating the tangents of the half angles of a triangle when the sides are given. The dihedral angles at A are X, B, C ; at B are A, Y, C ; and at C are A, B, Z ; and are to be found in the same way from the angles with suffixes 2, 3, 4 respectively. Thus the volume is calculated from four different expressions, and each dihedral angle from two; and the agreement of the results is sufficient to guarantee the correctness of the calculations. From each tetrahedron we get the six parts of each of four plane triangles, and the six parts of each of four spherical triangles; and these may be used to supply any deficiency in any of the preceding sets of examples; or for practice in solving spherical triangles. To make these plane and spherical triangles complete, I have given always the values of all the plane angles of each face of every tetrahedron, or rather the halves of these angles. The order of magnitude of $A + X, B + Y, C + Z$ being always the same as that of $a + x, b + y, c + z$; that of $A - X, B - Y, C - Z$, the same as that of $a - x, b - y, c - z$; that of $A - X, Y - B, Z - C$, the same as that of $a - x, y - b, z - c$, etc.; the results should be tested accordingly. When

two terms of one system of lengths are equal, the two corresponding terms of the corresponding system of angles will be also equal. In many cases, the data will be found to satisfy one, or two, of such equations, and it will be seen that the calculated angles satisfy the corresponding equations, *quam prox.* Many other properties of tetrahedra are illustrated, attention being directed to such cases by a very brief entry in column of remarks. The tetrahedron in which a, b, c are the lengths of three conterminous edges, and x, y, z the lengths of the edges respectively opposite

to a, b, c , is here denoted by $\left\{ \begin{array}{l} a \ x \\ b \ y \\ c \ z \end{array} \right\}$. The same tetrahedron will thus be also

denoted by $\left\{ \begin{array}{l} a \ x \\ y \ b \\ z \ c \end{array} \right\}$, $\left\{ \begin{array}{l} x \ a \\ b \ y \\ c \ z \end{array} \right\}$, or $\left\{ \begin{array}{l} x \ a \\ y \ b \\ c \ z \end{array} \right\}$. When six lengths are given, there will

be thirty different tetrahedra having these for edges, but some of these may be impossible, even when any set of three of the lengths are sides of a real triangle. In dealing with such a system of tetrahedra, it seems most convenient to arrange them in five groups of six, the longest edge having the same edge opposite to it in all the tetrahedra of any one group.

The sums of the dihedral angles at O, A, B, C respectively are denoted by $2S_1, 2S_2, 2S_3, 2S_4$; and the order of magnitude of S_1, S_2, S_3, S_4 corresponds to that of s_1, s_2, s_3, s_4 . When $a - x = b - y = c - z$, $s_2 = s_3 = s_4$, $S_2 = S_3 = S_4$; and, if $\sigma_2 > \sigma_3 > \sigma_4$, it will be observed that

$$\begin{aligned} \sin \sigma_2 \sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) &< \sin \sigma_3 \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3) \\ &< \sin \sigma_4 \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4); \end{aligned}$$

also

$$\begin{aligned} \sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) / \sin \sigma_2 &< \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3) / \sin \sigma_3 \\ &< \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4) / \sin \sigma_4. \end{aligned}$$

Many cases of maximum volume under certain conditions will be found.

From the equations of which $\frac{\partial V}{\partial a} = \frac{1}{8}ax \cot A$ is the type, we see that when

a, x vary subject to the condition that $a + x$ is constant, the volume (V) will be a maximum when $A = X$; when subject to the condition that $a - x$ is constant, V will be a maximum when $A + X = 180^\circ$; and similarly with the sum or difference of any pair of opposite edges. When $a, x, b + y, c + z$ are given, V will be a maximum when $B = Y, C = Z$, which involves $b = y, c = z$. When $a, x, b + y, c - z$ are given, V will be a maximum when $B = Y, C + Z = 180^\circ$. As the lengths of the edges are varied so as to approach this maximum value, it appears that the variations of b and y do not alter $C + Z$; nor those of c and z affect $B - Y$. When $a, x, b - y, c - z$ are given, it should seem that V will be a maximum when $B + Y = C + Z = 180^\circ$, but this maximum will be when b, y, c, z are infinite, and consequently V .

In any tetrahedron in which $\alpha_1 + \beta_1 + \gamma_1 = \alpha_2 + \beta_2 + \gamma_2 = \pi$, it will be found that

$$\begin{aligned} \gamma_1 + \alpha_1 - \beta_1 &= \gamma_4 + \alpha_4 - \beta_4; & \alpha_1 + \beta_1 - \gamma_1 &= \alpha_3 + \beta_3 - \gamma_3; \\ \gamma_2 + \alpha_2 - \beta_2 &= \gamma_3 + \alpha_3 - \beta_3; & \alpha_2 + \beta_2 - \gamma_2 &= \alpha_4 + \beta_4 - \gamma_4; \end{aligned}$$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
71. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 4' 19.49'' \\ 20^\circ 49' 36.58'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 33^\circ 10' 59.37'' \\ 34^\circ 14' 11.99'' \\ 22^\circ 25' 39.39'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 34^\circ 11' 57.06'' \\ 24^\circ 17' 59.87'' \end{cases}$	$\begin{cases} 46^\circ 44' 44.03'' \\ 43^\circ 37' 40.64'' \\ 22^\circ 37' 3.57'' \end{cases}$	$\begin{cases} 48^\circ 49' 59.22'' \\ 49^\circ 19' 16.21'' \\ 24^\circ 22' 49.11'' \end{cases}$	$\begin{cases} 38^\circ 48' 19.55'' \\ 34^\circ 22' 45.17'' \\ 22^\circ 24' 48.43'' \end{cases}$	13.77206	
72. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 4' 19.49'' \\ 14^\circ 18' 6.32'' \\ 19^\circ 59' 26.14'' \end{cases}$	$\begin{cases} 33^\circ 10' 59.37'' \\ 32^\circ 36' 52.49'' \\ 20^\circ 51' 11.06'' \end{cases}$	$\begin{cases} 37^\circ 23' 41.37'' \\ 22^\circ 37' 3.57'' \\ 24^\circ 17' 59.87'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 43^\circ 37' 40.64'' \\ 34^\circ 11' 57.06'' \end{cases}$	$\begin{cases} 56^\circ 13' 19.51'' \\ 39^\circ 20' 13.50'' \\ 22^\circ 57' 49.36'' \end{cases}$	$\begin{cases} 40^\circ 48' 28.45'' \\ 21^\circ 25' 25.05'' \\ 32^\circ 56' 27.75'' \end{cases}$	12.84365	
73. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 22^\circ 6' 5.66'' \\ 22^\circ 25' 39.39'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 31^\circ 26' 51.14'' \\ 34^\circ 14' 11.99'' \\ 20^\circ 49' 36.58'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 36^\circ 44' 30.25'' \\ 22^\circ 31' 9.05'' \end{cases}$	$\begin{cases} 46^\circ 44' 44.03'' \\ 45^\circ 22' 44.69'' \\ 21^\circ 48' 38.61'' \end{cases}$	$\begin{cases} 47^\circ 7' 58.93'' \\ 43^\circ 42' 58.08'' \\ 24^\circ 26' 12.81'' \end{cases}$	$\begin{cases} 36^\circ 27' 59.09'' \\ 37^\circ 37' 46.06'' \\ 21^\circ 47' 8.56'' \end{cases}$	13.47945	
74. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 22^\circ 6' 5.66'' \\ 13^\circ 34' 58.03'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 31^\circ 26' 51.14'' \\ 32^\circ 7' 34.19'' \\ 18^\circ 21' 32.66'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 21^\circ 48' 38.61'' \\ 22^\circ 31' 9.05'' \end{cases}$	$\begin{cases} 58^\circ 3' 29.31'' \\ 45^\circ 22' 44.69'' \\ 30^\circ 44' 30.25'' \end{cases}$	$\begin{cases} 58^\circ 32' 26.51'' \\ 41^\circ 15' 54.61'' \\ 20^\circ 34' 45.94'' \end{cases}$	$\begin{cases} 39^\circ 14' 51.79'' \\ 19^\circ 57' 32.53'' \\ 35^\circ 51' 56.23'' \end{cases}$	12.03492	(71) to (84) are also tetrahedra with the same six lengths for their edges. The six tetrahedra in which the two longest edges are opposite each other are all impossible.
75. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 20^\circ 53' 38.77'' \\ 20^\circ 51' 11.06'' \\ 19^\circ 59' 26.14'' \end{cases}$	$\begin{cases} 28^\circ 30' 37.55'' \\ 32^\circ 36' 52.49'' \\ 14^\circ 18' 6.32'' \end{cases}$	$\begin{cases} 37^\circ 23' 41.37'' \\ 32^\circ 18' 35.14'' \\ 15^\circ 32' 49.76'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 53^\circ 33' 31.47'' \\ 29^\circ 10' 47.31'' \end{cases}$	$\begin{cases} 51^\circ 28' 55.49'' \\ 49^\circ 13' 2.84'' \\ 15^\circ 43' 5.90'' \end{cases}$	$\begin{cases} 33^\circ 42' 51.86'' \\ 27^\circ 58' 17.77'' \\ 30^\circ 53' 33.48'' \end{cases}$	11.86436	
76. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 20^\circ 53' 38.77'' \\ 18^\circ 21' 32.66'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 28^\circ 30' 37.55'' \\ 32^\circ 7' 34.19'' \\ 13^\circ 34' 58.03'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 29^\circ 10' 47.31'' \\ 15^\circ 32' 49.76'' \end{cases}$	$\begin{cases} 58^\circ 3' 29.31'' \\ 53^\circ 33' 31.47'' \\ 32^\circ 18' 35.14'' \end{cases}$	$\begin{cases} 55^\circ 39' 49.04'' \\ 48^\circ 44' 29.87'' \\ 15^\circ 5' 6.60'' \end{cases}$	$\begin{cases} 33^\circ 42' 51.86'' \\ 27^\circ 58' 17.77'' \\ 30^\circ 53' 33.48'' \end{cases}$	11.34094	
77. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 28' 7.17'' \\ 14^\circ 18' 6.32'' \\ 19^\circ 40' 57.73'' \end{cases}$	$\begin{cases} 34^\circ 11' 57.06'' \\ 32^\circ 43' 41.30'' \\ 20^\circ 51' 11.06'' \end{cases}$	$\begin{cases} 37^\circ 35' 20.97'' \\ 30^\circ 17' 43.87'' \\ 24^\circ 16' 36.86'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 43^\circ 15' 15.97'' \\ 33^\circ 10' 59.37'' \end{cases}$	$\begin{cases} 56^\circ 38' 17.14'' \\ 38^\circ 26' 2.57'' \\ 22^\circ 42' 35.79'' \end{cases}$	$\begin{cases} 42^\circ 14' 54.68'' \\ 21^\circ 12' 50.88'' \\ 31^\circ 40' 45.14'' \end{cases}$	12.7562	
78. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7557, \end{cases}$	$\begin{cases} 22^\circ 31' 9.65'' \\ 22^\circ 23' 55.02'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 32^\circ 1' 22.37'' \\ 34^\circ 14' 11.92'' \\ 20^\circ 25' 47.66'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 36^\circ 17' 43.87'' \\ 22^\circ 6' 5.66'' \end{cases}$	$\begin{cases} 47^\circ 0' 17.32'' \\ 43^\circ 15' 15.97'' \\ 21^\circ 40' 53.76'' \end{cases}$	$\begin{cases} 47^\circ 45' 45.65'' \\ 45^\circ 22' 44.69'' \\ 22^\circ 17' 30.93'' \end{cases}$	$\begin{cases} 37^\circ 22' 19.32'' \\ 36^\circ 56' 27.33'' \\ 31^\circ 38' 0.51'' \end{cases}$	13.39033	
79. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7557, \end{cases}$	$\begin{cases} 22^\circ 31' 9.65'' \\ 13^\circ 22' 3.20'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 32^\circ 1' 22.37'' \\ 32^\circ 7' 34.19'' \\ 17^\circ 44' 30.19'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 21^\circ 40' 53.76'' \\ 22^\circ 6' 5.66'' \end{cases}$	$\begin{cases} 58^\circ 53' 26.61'' \\ 45^\circ 22' 44.69'' \\ 36^\circ 17' 43.87'' \end{cases}$	$\begin{cases} 59^\circ 29' 8.08'' \\ 40^\circ 42' 2.21'' \\ 19^\circ 47' 31.02'' \end{cases}$	$\begin{cases} 40^\circ 22' 54.14'' \\ 19^\circ 26' 3.19'' \\ 35^\circ 19' 41.99'' \end{cases}$	11.7694	
80. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 21^\circ 28' 4.14'' \\ 20^\circ 51' 11.06'' \\ 19^\circ 40' 57.73'' \end{cases}$	$\begin{cases} 20^\circ 10' 47.31'' \\ 32^\circ 43' 41.30'' \\ 14^\circ 18' 6.32'' \end{cases}$	$\begin{cases} 37^\circ 35' 20.97'' \\ 32^\circ 18' 35.14'' \\ 15^\circ 40' 18.85'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 52^\circ 31' 37.01'' \\ 28^\circ 30' 37.55'' \end{cases}$	$\begin{cases} 51^\circ 47' 4.39'' \\ 48^\circ 10' 54.29'' \\ 15^\circ 46' 7.56'' \end{cases}$	$\begin{cases} 34^\circ 13' 46.68'' \\ 32^\circ 38' 55.92'' \\ 30^\circ 0' 14.77'' \end{cases}$	11.9202	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
91. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=5.3856, \end{cases}$ $\begin{cases} x=6.8205 \\ y=4.7557 \\ z=4.8265 \end{cases}$	$\begin{cases} 40^\circ 27' 9.94'' \\ 20^\circ 29' 50.39'' \\ 20^\circ 49' 36.58'' \end{cases}$ $\begin{cases} 45^\circ 22' 44.69'' \\ 22^\circ 25' 39.49'' \\ 23^\circ 59' 18.49'' \end{cases}$	$\begin{cases} 46^\circ 44' 44.03'' \\ 22^\circ 6' 5.66'' \\ 25^\circ 36' 58.48'' \end{cases}$ $\begin{cases} 37^\circ 31' 4.45'' \\ 26^\circ 44' 41.79'' \\ 26^\circ 54' 1.45'' \end{cases}$	$\begin{cases} 45^\circ 30' 51.12'' \\ 23^\circ 55' 51.58'' \\ 22^\circ 31' 9.65'' \end{cases}$ $\begin{cases} 79^\circ 50' 21.87'' \\ 7^\circ 27' 45.82'' \\ 7^\circ 52' 12.49'' \end{cases}$	5.395218	See previous page.			
92. $\begin{cases} a=7.3871, \\ b=5.9635, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.9870 \\ y=5.9634 \\ z=5.7634 \end{cases}$	$\begin{cases} 35^\circ 10' 9.55'' \\ 25^\circ 37' 31.80'' \\ 24^\circ 52' 47.74'' \end{cases}$ $\begin{cases} 36^\circ 33' 34.19'' \\ 26^\circ 5' 8.05'' \\ 26^\circ 51' 23.75'' \end{cases}$	$\begin{cases} 39^\circ 2' 4.21'' \\ 22^\circ 22' 44.36'' \\ 28^\circ 5' 8.66'' \end{cases}$ $\begin{cases} 37^\circ 31' 4.45'' \\ 26^\circ 44' 41.79'' \\ 26^\circ 54' 1.45'' \end{cases}$	$\begin{cases} 48^\circ 16' 23.66'' \\ 27^\circ 40' 55.35'' \\ 26^\circ 49' 13.82'' \end{cases}$ $\begin{cases} 44^\circ 58' 17.27'' \\ 29^\circ 36' 31.67'' \\ 28^\circ 43' 27.66'' \end{cases}$	26.76496				
93. $\begin{cases} a=7.1471, \\ b=5.9669, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.7470 \\ y=5.9600 \\ z=5.7634 \end{cases}$	$\begin{cases} 33^\circ 46' 57.23'' \\ 26^\circ 17' 5.52'' \\ 25^\circ 35' 23.03'' \end{cases}$ $\begin{cases} 35^\circ 7' 27.48'' \\ 26^\circ 53' 1.12'' \\ 27^\circ 36' 10.53'' \end{cases}$	$\begin{cases} 37^\circ 31' 35.85'' \\ 28^\circ 7' 14.44'' \\ 28^\circ 48' 9.39'' \end{cases}$ $\begin{cases} 46^\circ 37' 29.15'' \\ 29^\circ 29' 35.41'' \\ 30^\circ 22' 39.48'' \end{cases}$	27.04664	(92) to (96) exemplify the process of finding the tetrahedron of maximum volume when $a-x$, $b+y$, c , and z are given.				
94. $\begin{cases} a=7.0871, \\ b=5.9769, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.6870 \\ y=5.9500 \\ z=5.7634 \end{cases}$	$\begin{cases} 33^\circ 24' 43.34'' \\ 26^\circ 24' 9.16'' \\ 25^\circ 45' 7.03'' \end{cases}$ $\begin{cases} 34^\circ 48' 5.32'' \\ 27^\circ 7' 42.21'' \\ 27^\circ 48' 9.75'' \end{cases}$	$\begin{cases} 37^\circ 7' 10.76'' \\ 28^\circ 15' 20.69'' \\ 28^\circ 57' 40.82'' \end{cases}$ $\begin{cases} 45^\circ 52' 4.32'' \\ 29^\circ 58' 33.44'' \\ 30^\circ 49' 22.42'' \end{cases}$	27.11766					
95. $\begin{cases} a=7.0841, \\ b=5.9807, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.6840 \\ y=5.9462 \\ z=5.7634 \end{cases}$	$\begin{cases} 33^\circ 23' 1.17'' \\ 26^\circ 23' 32.04'' \\ 25^\circ 45' 19.88'' \end{cases}$ $\begin{cases} 34^\circ 47' 49.00'' \\ 27^\circ 9' 27.60'' \\ 27^\circ 49' 4.82'' \end{cases}$	$\begin{cases} 37^\circ 5' 12.52'' \\ 28^\circ 14' 41.88'' \\ 28^\circ 57' 44.80'' \end{cases}$ $\begin{cases} 45^\circ 49' 45.62'' \\ 30^\circ 0' 48.71'' \\ 30^\circ 50' 36.71'' \end{cases}$	27.117785					
96. $\begin{cases} a=7.084115, \\ b=5.980813, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.684015 \\ y=5.946087 \\ z=5.7634 \end{cases}$	$\begin{cases} 33^\circ 23' 0.26'' \\ 26^\circ 23' 29.90'' \\ 25^\circ 45' 19.18'' \end{cases}$ $\begin{cases} 34^\circ 47' 50.76'' \\ 27^\circ 9' 29.52'' \\ 27^\circ 49' 5.29'' \end{cases}$	$\begin{cases} 37^\circ 5' 11.36'' \\ 28^\circ 14' 39.56'' \\ 28^\circ 57' 43.80'' \end{cases}$ $\begin{cases} 45^\circ 49' 46.14'' \\ 30^\circ 0' 50.02'' \\ 30^\circ 50' 36.16'' \end{cases}$	27.117785					
97. $\begin{cases} a=4.886136, \\ b=3.967768, \\ c=4.3257, \end{cases}$ $\begin{cases} x=3.775036 \\ y=3.572532 \\ z=3.2146 \end{cases}$	$\begin{cases} 26^\circ 58' 18.90'' \\ 22^\circ 33' 53.45'' \\ 20^\circ 28' 36.21'' \end{cases}$ $\begin{cases} 33^\circ 40' 27.35'' \\ 26^\circ 59' 59.41'' \\ 29^\circ 33' 10.32'' \end{cases}$	$\begin{cases} 37^\circ 52' 56.23'' \\ 29^\circ 5' 25.22'' \\ 25^\circ 54' 0.02'' \end{cases}$ $\begin{cases} 48^\circ 26' 47.83'' \\ 30^\circ 14' 47.56'' \\ 33^\circ 41' 35.39'' \end{cases}$	6.445679	$A-X=C-Z$, $B+Y=C+Z$, $A+X=180^\circ$, $B-Y=0^\circ$, a similar case of maximum volume.				
98. $\begin{cases} a=5.3147, \\ b=4.0257, \\ c=4.3257, \end{cases}$ $\begin{cases} x=4.2036 \\ y=3.5146 \\ z=3.2146 \end{cases}$	$\begin{cases} 30^\circ 9' 27.67'' \\ 20^\circ 35' 27.40'' \\ 18^\circ 33' 42.30'' \end{cases}$ $\begin{cases} 38^\circ 35' 14.65'' \\ 24^\circ 32' 54.36'' \\ 27^\circ 4' 4.68'' \end{cases}$	$\begin{cases} 46^\circ 53' 23.34'' \\ 27^\circ 18' 18.90'' \\ 31^\circ 41' 24.26'' \end{cases}$ $\begin{cases} 58^\circ 15' 33.72'' \\ 23^\circ 29' 14.39'' \\ 21^\circ 21' 1.89'' \end{cases}$	5.912593	$A-X=C-Z$; $B+Y=C+Z$				
99. $\begin{cases} a=11.99045, \\ b=12.0002, \\ c=15.01087, \end{cases}$ $\begin{cases} x=10.96695 \\ y=9.7858 \\ z=10.70947 \end{cases}$	$\begin{cases} 23^\circ 7' 58.61'' \\ 20^\circ 17' 26.61'' \\ 26^\circ 30' 47.00'' \end{cases}$ $\begin{cases} 32^\circ 15' 47.00'' \\ 31^\circ 46' 0.51'' \\ 43^\circ 16' 16.71'' \end{cases}$	$\begin{cases} 31^\circ 43' 12.40'' \\ 26^\circ 49' 11.31'' \\ 40^\circ 44' 41.19'' \end{cases}$ $\begin{cases} 31^\circ 35' 16.82'' \\ 31^\circ 7' 10.35'' \\ 49^\circ 40' 40.22'' \end{cases}$	166.9554	Tetrahedron of max. vol. when $a+x$, b , y , and $c-z$ are given.				
100. $\begin{cases} a=5.966907, \\ b=6.028293, \\ c=7.5063, \end{cases}$ $\begin{cases} x=5.4836 \\ y=4.8914 \\ z=4.3556 \end{cases}$	$\begin{cases} 23^\circ 6' 38.24'' \\ 20^\circ 17' 45.64'' \\ 21^\circ 17' 22.23'' \end{cases}$ $\begin{cases} 36^\circ 14' 24.17'' \\ 34^\circ 43' 53.23'' \\ 43^\circ 26' 9.60'' \end{cases}$	$\begin{cases} 34^\circ 43' 13.30'' \\ 34^\circ 43' 13.30'' \\ 46^\circ 38' 17.92'' \end{cases}$ $\begin{cases} 36^\circ 13' 46.82'' \\ 26^\circ 36' 59.18'' \\ 31^\circ 39' 31.09'' \end{cases}$	18.889605	$a=B$, $A=B$				

EXAMPLES. X. — Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}X$, $\frac{1}{2}Y$, $\frac{1}{2}Z$	VOLUME.	REMARKS.
101. $\begin{cases} a=6.4327, \\ b=6.12078, \\ c=5.7511, \end{cases}$	$\begin{cases} 11^{\circ}14'51.07'' \\ 12^{\circ}10'51.59'' \\ 13^{\circ}52'27.06'' \end{cases}$	$\begin{cases} 24^{\circ}5'44.22'' \\ 35^{\circ}11'26.38'' \\ 31^{\circ}38'41.45'' \end{cases}$	$\begin{cases} 40^{\circ}56'6.56'' \\ 28^{\circ}48'46.76'' \\ 34^{\circ}55'35.82'' \end{cases}$	$\begin{cases} 46^{\circ}10'26.96'' \\ 43^{\circ}49'33.11'' \\ 37^{\circ}5'29.02'' \end{cases}$	$\begin{cases} 46^{\circ}33'16.62'' \\ 43^{\circ}26'43.38'' \\ 35^{\circ}37'35.68'' \end{cases}$	$\begin{cases} 26^{\circ}5'58.58'' \\ 29^{\circ}12'31.82'' \\ 37^{\circ}1'39.52'' \end{cases}$	$\begin{cases} A+B=180^{\circ} \\ A+X=B+Y=C+Z \end{cases}$	5.727094	
102. $\begin{cases} a=6.4317, \\ b=7.275188, \\ c=3.3423, \end{cases}$	$\begin{cases} 30^{\circ}47'57.07'' \\ 29^{\circ}52'37.47'' \\ 13^{\circ}32'7.44'' \end{cases}$	$\begin{cases} 44^{\circ}18'59.64'' \\ 45^{\circ}24'6.91'' \\ 15^{\circ}39'6.36'' \end{cases}$	$\begin{cases} 31^{\circ}3'45.65'' \\ 30^{\circ}6'42.54'' \\ 13^{\circ}40'19.30'' \end{cases}$	$\begin{cases} 44^{\circ}28'16.17'' \\ 45^{\circ}31'43.63'' \\ 15^{\circ}34'17.82'' \end{cases}$	$\begin{cases} 43^{\circ}6'2.71'' \\ 46^{\circ}53'57.29'' \\ 15^{\circ}36'54.82'' \end{cases}$	$\begin{cases} 43^{\circ}1'19.42'' \\ 39^{\circ}13'24.84'' \\ 15^{\circ}33'11.53'' \end{cases}$	$\begin{cases} a_1+\beta_1=180^{\circ}=A+B; \\ A+\lambda=B+Y; \\ A-\lambda=C-Z \end{cases}$	10.22294	
103. $\begin{cases} a=5.8341, \\ b=5.2931, \\ c=6.3758, \end{cases}$	$\begin{cases} 13^{\circ}37'59.26'' \\ 16^{\circ}23'50.93'' \\ 18^{\circ}3'12.12'' \end{cases}$	$\begin{cases} 24^{\circ}59'0.56'' \\ 31^{\circ}43'53.01'' \\ 41^{\circ}5'17.14'' \end{cases}$	$\begin{cases} 40^{\circ}12'54.87'' \\ 32^{\circ}30'7.51'' \\ 48^{\circ}39'3.07'' \end{cases}$	$\begin{cases} 32^{\circ}50'51.93'' \\ 27^{\circ}42'57.67'' \\ 32^{\circ}30'51.93'' \end{cases}$	$\begin{cases} 37^{\circ}55'8.72'' \\ 30^{\circ}52'1.71'' \\ 51^{\circ}22'41.57'' \end{cases}$	$\begin{cases} 24^{\circ}27'35.87'' \\ 31^{\circ}30'42.88'' \\ 37^{\circ}55'8.72'' \end{cases}$	$\begin{cases} a_1=74, A=Z, \\ A+\lambda=B+Y, \\ A-\lambda=C-Z \end{cases}$	7.895052	
104. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\begin{cases} 22^{\circ}29'33.12'' \\ 22^{\circ}29'33.12'' \\ 22^{\circ}29'33.12'' \end{cases}$	$\begin{cases} 28^{\circ}27'41.63'' \\ 33^{\circ}27'45.32'' \\ 23^{\circ}46'13.48'' \end{cases}$	$\begin{cases} 34^{\circ}2'41.56'' \\ 28^{\circ}49'4.04'' \\ 24^{\circ}1'21.89'' \end{cases}$	$\begin{cases} 43^{\circ}44'13.40'' \\ 43^{\circ}29'4.99'' \\ 32^{\circ}43'14.33'' \end{cases}$	$\begin{cases} 44^{\circ}18'10.63'' \\ 43^{\circ}54'37.65'' \\ 26^{\circ}37'44.26'' \end{cases}$	$\begin{cases} 32^{\circ}45'47.19'' \\ 32^{\circ}45'47.19'' \\ 32^{\circ}45'47.19'' \end{cases}$	$\begin{cases} a_1=\beta_1=1 \\ X=Y=Z \end{cases}$	10.16938	
105. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893, \end{cases}$	$\begin{cases} 22^{\circ}24'5.52'' \\ 22^{\circ}27'24.92'' \\ 21^{\circ}45'47.46'' \end{cases}$	$\begin{cases} 28^{\circ}57'29.27'' \\ 33^{\circ}48'59.21'' \\ 24^{\circ}27'0.33'' \end{cases}$	$\begin{cases} 34^{\circ}35'13.32'' \\ 29^{\circ}24'59.22'' \\ 24^{\circ}42'44.07'' \end{cases}$	$\begin{cases} 43^{\circ}5'34.75'' \\ 42^{\circ}53'10.41'' \\ 31^{\circ}37'31.51'' \end{cases}$	$\begin{cases} 43^{\circ}55'38.20'' \\ 43^{\circ}35'30.66'' \\ 27^{\circ}14'19.72'' \end{cases}$	$\begin{cases} 33^{\circ}6'35.14'' \\ 33^{\circ}14'12.69'' \\ 31^{\circ}42'42.87'' \end{cases}$	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.	10.17433	
106. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893, \end{cases}$	$\begin{cases} 22^{\circ}24'5.52'' \\ 23^{\circ}37'55.51'' \\ 20^{\circ}48'36.44'' \end{cases}$	$\begin{cases} 28^{\circ}57'29.27'' \\ 34^{\circ}16'39.91'' \\ 24^{\circ}20'44.08'' \end{cases}$	$\begin{cases} 34^{\circ}54'43.65'' \\ 31^{\circ}37'31.51'' \\ 24^{\circ}42'44.07'' \end{cases}$	$\begin{cases} 42^{\circ}1'20.41'' \\ 42^{\circ}53'10.41'' \\ 29^{\circ}24'59.22'' \end{cases}$	$\begin{cases} 42^{\circ}47'43.69'' \\ 44^{\circ}19'33.48'' \\ 26^{\circ}53'33.39'' \end{cases}$	$\begin{cases} 32^{\circ}45'19.12'' \\ 35^{\circ}46'12.50'' \\ 29^{\circ}31'58.52'' \end{cases}$		10.15133	
107. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\begin{cases} 22^{\circ}28'41.68'' \\ 24^{\circ}27'0.33'' \\ 21^{\circ}45'47.46'' \end{cases}$	$\begin{cases} 27^{\circ}58'4.26'' \\ 33^{\circ}48'59.21'' \\ 22^{\circ}27'24.92'' \end{cases}$	$\begin{cases} 34^{\circ}25'13.33'' \\ 31^{\circ}37'17.14'' \\ 22^{\circ}45'46.37'' \end{cases}$	$\begin{cases} 43^{\circ}5'34.75'' \\ 44^{\circ}45'31.99'' \\ 30^{\circ}24'38.60'' \end{cases}$	$\begin{cases} 42^{\circ}57'0.90'' \\ 45^{\circ}47'23.14'' \\ 24^{\circ}52'23.86'' \end{cases}$	$\begin{cases} 31^{\circ}47'2.89'' \\ 36^{\circ}22'31.88'' \\ 30^{\circ}23'35.00'' \end{cases}$		9.9560565	
108. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\begin{cases} 22^{\circ}28'41.68'' \\ 23^{\circ}46'13.46'' \\ 22^{\circ}17'0.28'' \end{cases}$	$\begin{cases} 27^{\circ}58'4.26'' \\ 33^{\circ}33'50.81'' \\ 22^{\circ}29'33.12'' \end{cases}$	$\begin{cases} 34^{\circ}9'8.91'' \\ 30^{\circ}24'38.60'' \\ 22^{\circ}45'46.37'' \end{cases}$	$\begin{cases} 43^{\circ}44'13.42'' \\ 44^{\circ}45'31.99'' \\ 30^{\circ}24'38.60'' \end{cases}$	$\begin{cases} 43^{\circ}42'25.74'' \\ 45^{\circ}22'0.83'' \\ 25^{\circ}3'2.82'' \end{cases}$	$\begin{cases} 32^{\circ}0'54.51'' \\ 34^{\circ}55'4.75'' \\ 31^{\circ}37'7.61'' \end{cases}$		9.971434	
109. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\begin{cases} 22^{\circ}26'30.38'' \\ 22^{\circ}29'33.12'' \\ 22^{\circ}17'0.28'' \end{cases}$	$\begin{cases} 28^{\circ}33'46.61'' \\ 33^{\circ}33'50.81'' \\ 23^{\circ}46'13.46'' \end{cases}$	$\begin{cases} 34^{\circ}9'8.91'' \\ 20^{\circ}0'25.82'' \\ 24^{\circ}1'31.91'' \end{cases}$	$\begin{cases} 43^{\circ}44'13.42'' \\ 43^{\circ}31'57.71'' \\ 32^{\circ}25'47.57'' \end{cases}$	$\begin{cases} 44^{\circ}17'33.55'' \\ 43^{\circ}58'16.11'' \\ 26^{\circ}34'27.95'' \end{cases}$	$\begin{cases} 32^{\circ}49'5.26'' \\ 32^{\circ}55'51.85'' \\ 32^{\circ}28'14.48'' \end{cases}$		10.103635	
110. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\begin{cases} 22^{\circ}26'30.38'' \\ 24^{\circ}20'44.08'' \\ 20^{\circ}48'36.44'' \end{cases}$	$\begin{cases} 28^{\circ}33'46.61'' \\ 34^{\circ}16'39.91'' \\ 23^{\circ}37'55.51'' \end{cases}$	$\begin{cases} 34^{\circ}54'43.65'' \\ 32^{\circ}25'47.57'' \\ 24^{\circ}1'31.91'' \end{cases}$	$\begin{cases} 43^{\circ}1'20.41'' \\ 43^{\circ}31'57.71'' \\ 29^{\circ}0'25.82'' \end{cases}$	$\begin{cases} 42^{\circ}24'16.89'' \\ 45^{\circ}7'39.98'' \\ 26^{\circ}3'4.63'' \end{cases}$	$\begin{cases} 32^{\circ}13'57.17'' \\ 32^{\circ}55'14.60'' \\ 29^{\circ}4'1.39'' \end{cases}$	In (110), (113) the volumes are equal, and the values of B supplementary.	10.06527	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}A_1, \frac{1}{2}B_1, \frac{1}{2}C_1$	$\frac{1}{2}A_2, \frac{1}{2}B_2, \frac{1}{2}C_2$	$\frac{1}{2}A_3, \frac{1}{2}B_3, \frac{1}{2}C_3$	$\frac{1}{2}A_4, \frac{1}{2}B_4, \frac{1}{2}C_4$	$\frac{1}{2}A_5, \frac{1}{2}B_5, \frac{1}{2}C_5$	$\frac{1}{2}A_6, \frac{1}{2}B_6, \frac{1}{2}C_6$	$\frac{1}{2}A_7, \frac{1}{2}B_7, \frac{1}{2}C_7$	$\frac{1}{2}A_8, \frac{1}{2}B_8, \frac{1}{2}C_8$	$\frac{1}{2}A_9, \frac{1}{2}B_9, \frac{1}{2}C_9$	$\frac{1}{2}A_{10}, \frac{1}{2}B_{10}, \frac{1}{2}C_{10}$	VOLUME.	REMARKS.
111. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.1941 \\ c=4.2893, & z=3.9794 \end{cases}$	$22^\circ 39' 55.74''$ $23^\circ 51' 55.51''$ $20^\circ 35' 48.29''$	$29^\circ 24' 59.22''$ $34^\circ 22' 51.05''$ $24^\circ 20' 44.08''$	$35^\circ 1' 20.66''$ $31^\circ 37' 31.51''$ $24^\circ 41' 24.64''$	$42^\circ 1' 20.41''$ $42^\circ 38' 39.62''$ $28^\circ 57' 29.27''$	$42^\circ 57' 30.25''$ $44^\circ 4' 51.88''$ $26^\circ 49' 42.42''$	$33^\circ 17' 4.01''$ $35^\circ 41' 10.06''$ $29^\circ 5' 23.85''$	IO.14892					
112. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.1941 \\ c=4.2893, & z=3.9794 \end{cases}$	$22^\circ 39' 55.74''$ $22^\circ 11' 35.74''$ $21^\circ 45' 47.46''$	$29^\circ 24' 59.22''$ $33^\circ 48' 59.21''$ $24^\circ 28' 3.70''$	$35^\circ 1' 20.66''$ $28^\circ 57' 29.27''$ $24^\circ 41' 24.64''$	$42^\circ 1' 20.41''$ $42^\circ 38' 39.62''$ $31^\circ 37' 31.51''$	$42^\circ 57' 30.25''$ $44^\circ 4' 51.88''$ $26^\circ 49' 42.42''$	$33^\circ 17' 4.01''$ $35^\circ 41' 10.06''$ $29^\circ 5' 23.85''$	IO.17192					
113. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.2893 \\ c=4.1941, & z=3.9794 \end{cases}$	$22^\circ 42' 40.72''$ $24^\circ 20' 44.08''$ $20^\circ 35' 48.29''$	$29^\circ 0' 25.82''$ $34^\circ 22' 51.05''$ $23^\circ 27' 55.51''$	$35^\circ 1' 20.66''$ $32^\circ 25' 47.57''$ $24^\circ 0' 35.31''$	$42^\circ 1' 20.41''$ $42^\circ 38' 39.62''$ $28^\circ 57' 29.27''$	$42^\circ 57' 30.25''$ $44^\circ 4' 51.88''$ $26^\circ 49' 42.42''$	$33^\circ 17' 4.01''$ $35^\circ 41' 10.06''$ $29^\circ 5' 23.85''$	IO.06527					
114. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.1941 \\ c=4.1941, & z=4.2893 \end{cases}$	$22^\circ 42' 40.72''$ $23^\circ 13' 23.52''$ $22^\circ 17' 0.28''$	$29^\circ 0' 25.82''$ $33^\circ 33' 50.81''$ $23^\circ 46' 53.71''$	$35^\circ 1' 20.66''$ $32^\circ 25' 47.57''$ $24^\circ 0' 35.31''$	$42^\circ 1' 20.41''$ $42^\circ 38' 39.62''$ $31^\circ 37' 31.51''$	$42^\circ 57' 30.25''$ $44^\circ 4' 51.88''$ $26^\circ 49' 42.42''$	$33^\circ 17' 4.01''$ $35^\circ 41' 10.06''$ $29^\circ 5' 23.85''$	IO.10116					
115. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.1941 \\ c=3.9794, & z=4.2893 \end{cases}$	$22^\circ 45' 46.37''$ $23^\circ 40' 53.71''$ $22^\circ 17' 0.28''$	$28^\circ 16' 10.80''$ $33^\circ 33' 50.81''$ $22^\circ 13' 23.52''$	$34^\circ 9' 8.91''$ $30^\circ 16' 2.76''$ $22^\circ 28' 41.68''$	$43^\circ 59' 42.77''$ $44^\circ 45' 31.95''$ $31^\circ 27' 46.44''$	$43^\circ 59' 41.32''$ $45^\circ 14' 34.29''$ $24^\circ 33' 54.85''$	$32^\circ 25' 12.21''$ $33^\circ 20' 2.08''$ $32^\circ 27' 50.67''$	9.94747					
116. $\begin{cases} a=5.6794, & x=4.0188 \\ b=5.6318, & y=4.2893 \\ c=3.9794, & z=4.1941 \end{cases}$	$22^\circ 45' 46.37''$ $24^\circ 28' 3.70''$ $21^\circ 45' 47.46''$	$28^\circ 16' 10.80''$ $33^\circ 48' 59.21''$ $22^\circ 11' 35.74''$	$34^\circ 9' 8.91''$ $30^\circ 16' 2.76''$ $22^\circ 28' 41.68''$	$43^\circ 59' 42.77''$ $44^\circ 45' 31.95''$ $31^\circ 27' 46.44''$	$43^\circ 59' 41.32''$ $45^\circ 14' 34.29''$ $24^\circ 33' 54.85''$	$32^\circ 25' 12.21''$ $33^\circ 20' 2.08''$ $32^\circ 27' 50.67''$	9.934566					
117. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=4.0188 \\ c=4.2893, & z=3.9794 \end{cases}$	$23^\circ 50' 32.39''$ $24^\circ 22' 4.92''$ $20^\circ 35' 48.29''$	$31^\circ 37' 31.51''$ $34^\circ 22' 51.05''$ $24^\circ 27' 0.33''$	$35^\circ 1' 20.66''$ $29^\circ 24' 59.22''$ $24^\circ 34' 0.17''$	$43^\circ 5' 34.75''$ $41^\circ 35' 27.44''$ $28^\circ 57' 29.27''$	$44^\circ 32' 55.77''$ $45^\circ 39' 43.55''$ $26^\circ 46' 57.36''$	$36^\circ 13' 5.68''$ $36^\circ 10' 42.75''$ $30^\circ 15' 20.49''$	IO.13696					
118. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=3.9794 \\ c=4.2893, & z=4.0188 \end{cases}$	$23^\circ 50' 32.39''$ $22^\circ 11' 35.74''$ $20^\circ 48' 36.44''$	$31^\circ 37' 31.51''$ $34^\circ 16' 39.91''$ $24^\circ 28' 3.70''$	$35^\circ 1' 20.66''$ $28^\circ 57' 29.27''$ $24^\circ 34' 0.17''$	$43^\circ 5' 34.75''$ $41^\circ 35' 27.44''$ $29^\circ 24' 59.22''$	$44^\circ 32' 55.77''$ $45^\circ 39' 43.55''$ $26^\circ 46' 57.36''$	$36^\circ 13' 5.68''$ $36^\circ 10' 42.75''$ $30^\circ 15' 20.49''$	IO.13696					
119. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=4.0188 \\ c=4.0188, & z=3.9794 \end{cases}$	$24^\circ 0' 35.31''$ $24^\circ 27' 0.33''$ $20^\circ 35' 48.29''$	$30^\circ 24' 38.60''$ $34^\circ 22' 51.05''$ $22^\circ 27' 24.92''$	$35^\circ 1' 20.66''$ $31^\circ 37' 31.51''$ $22^\circ 42' 40.72''$	$43^\circ 5' 34.75''$ $41^\circ 35' 27.44''$ $29^\circ 24' 59.22''$	$44^\circ 32' 55.77''$ $45^\circ 39' 43.55''$ $26^\circ 46' 57.36''$	$36^\circ 13' 5.68''$ $36^\circ 10' 42.75''$ $30^\circ 15' 20.49''$	9.956056					In (107), (119), (120) $V=V_2=V_3$ $A_1=B_3$ $A_2+A_5=180^\circ$ $B_1+B_2=180^\circ$
120. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=3.9794 \\ c=4.0188, & z=4.2893 \end{cases}$	$24^\circ 0' 35.31''$ $22^\circ 14' 22.20''$ $22^\circ 17' 0.28''$	$30^\circ 24' 38.60''$ $33^\circ 33' 50.81''$ $22^\circ 31' 9.76''$	$34^\circ 9' 8.91''$ $27^\circ 58' 4.26''$ $22^\circ 42' 40.72''$	$43^\circ 5' 34.75''$ $41^\circ 35' 27.44''$ $31^\circ 37' 31.51''$	$44^\circ 32' 55.77''$ $45^\circ 39' 43.55''$ $26^\circ 46' 57.36''$	$36^\circ 13' 5.68''$ $36^\circ 10' 42.75''$ $30^\circ 15' 20.49''$	9.956056					

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
121. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$24^\circ 1' 31.91''$ $22^\circ 31' 9.76''$ $22^\circ 17' 0.28''$	$30^\circ 16' 2.76''$ $33^\circ 33' 50.81''$ $22^\circ 14' 22.20''$	$34^\circ 9' 8.91''$ $28^\circ 16' 10.80''$ $22^\circ 26' 30.38''$	$45^\circ 14' 28.04''$ $43^\circ 13' 57.71''$ $31^\circ 27' 46.44''$	$46^\circ 1' 19.38''$ $43^\circ 13' 42.22''$ $24^\circ 41' 14.15''$	$35^\circ 17' 35.48''$ $31^\circ 54' 8.31''$ $31^\circ 25' 45.34''$	9.9345675	In (116), (121), (122) $V = V_1 = V_2 = V_3$ $B_1 + B_2 = 180^\circ$
122. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$24^\circ 1' 31.91''$ $24^\circ 28' 3.70''$ $20^\circ 48' 36.44''$	$30^\circ 16' 2.76''$ $34^\circ 16' 39.91''$ $22^\circ 11' 35.74''$	$34^\circ 54' 43.65''$ $31^\circ 27' 46.44''$ $22^\circ 26' 30.38''$	$43^\circ 20' 20.56''$ $43^\circ 31' 57.71''$ $28^\circ 16' 10.80''$	$43^\circ 58' 40.62''$ $44^\circ 20' 16.45''$ $24^\circ 21' 8.85''$	$34^\circ 37' 37.77''$ $35^\circ 43' 22.36''$ $28^\circ 18' 54.19''$	9.934567	In (117), (118) the volumes are equal and the values of $\mathcal{A}$ supplementary.
123. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$	$24^\circ 34' 0.17''$ $22^\circ 29' 33.12''$ $20^\circ 35' 48.29''$	$32^\circ 25' 47.57''$ $34^\circ 22' 57.05''$ $23^\circ 46' 13.46''$	$35^\circ 1' 20.66''$ $29^\circ 0' 25.82''$ $23^\circ 50' 32.39''$	$43^\circ 44' 13.42''$ $41^\circ 35' 27.44''$ $28^\circ 33' 46.61''$	$45^\circ 4' 59.26''$ $41^\circ 45' 33.12''$ $25^\circ 55' 38.77''$	$37^\circ 24' 0.02''$ $32^\circ 13' 1.31''$ $28^\circ 35' 34.23''$	10.04542	
124. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$	$24^\circ 34' 0.17''$ $22^\circ 13' 23.52''$ $20^\circ 48' 36.44''$	$32^\circ 25' 47.57''$ $34^\circ 16' 39.91''$ $23^\circ 46' 53.71''$	$34^\circ 54' 43.65''$ $28^\circ 33' 46.61''$ $23^\circ 50' 32.39''$	$43^\circ 59' 42.77''$ $41^\circ 35' 27.44''$ $29^\circ 0' 25.82''$	$45^\circ 56' 47.33''$ $41^\circ 36' 27.41''$ $25^\circ 58' 11.14''$	$37^\circ 29' 6.12''$ $31^\circ 42' 29.17''$ $29^\circ 0' 37.28''$	10.042935	
125. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=6.0188. \end{cases}$	$24^\circ 41' 24.64''$ $23^\circ 46' 13.46''$ $20^\circ 35' 48.29''$	$31^\circ 37' 17.11''$ $34^\circ 22' 51.05''$ $22^\circ 29' 33.12''$	$35^\circ 1' 20.66''$ $30^\circ 24' 38.60''$ $22^\circ 39' 55.74''$	$43^\circ 44' 13.42''$ $42^\circ 38' 39.62''$ $27^\circ 58' 4.26''$	$45^\circ 4' 1.75''$ $43^\circ 0' 35.00''$ $24^\circ 34' 58.35''$	$36^\circ 26' 19.21''$ $34^\circ 7' 47.34''$ $28^\circ 0' 33.33''$	9.9481965	
126. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.0188. \end{cases}$	$24^\circ 41' 24.64''$ $22^\circ 14' 22.20''$ $21^\circ 45' 47.46''$	$31^\circ 37' 17.14''$ $33^\circ 48' 59.21''$ $22^\circ 31' 9.76''$	$34^\circ 25' 13.33''$ $27^\circ 58' 4.26''$ $22^\circ 39' 55.74''$	$45^\circ 14' 28.04''$ $42^\circ 38' 39.62''$ $30^\circ 24' 38.60''$	$46^\circ 35' 46.61''$ $42^\circ 9' 56.36''$ $24^\circ 47' 39.52''$	$36^\circ 57' 24.70''$ $31^\circ 14' 39.59''$ $30^\circ 19' 59.15''$	9.932784	
127. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$24^\circ 42' 44.07''$ $22^\circ 31' 9.76''$ $21^\circ 45' 47.46''$	$31^\circ 27' 46.44''$ $33^\circ 48' 59.21''$ $22^\circ 14' 22.20''$	$34^\circ 25' 13.33''$ $28^\circ 16' 10.80''$ $22^\circ 24' 5.52''$	$45^\circ 14' 28.04''$ $42^\circ 53' 10.41''$ $30^\circ 16' 2.76''$	$46^\circ 28' 29.05''$ $44^\circ 26' 4.80''$ $24^\circ 29' 45.00''$	$36^\circ 46' 5.94''$ $31^\circ 39' 4.54''$ $30^\circ 12' 7.85''$	9.913712	
128. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$24^\circ 42' 44.07''$ $23^\circ 46' 53.71''$ $20^\circ 48' 36.44''$	$31^\circ 27' 46.44''$ $34^\circ 16' 39.91''$ $22^\circ 13' 23.52''$	$34^\circ 54' 43.65''$ $30^\circ 16' 2.76''$ $22^\circ 24' 5.52''$	$43^\circ 59' 42.77''$ $42^\circ 53' 10.41''$ $28^\circ 16' 10.80''$	$45^\circ 11' 31.56''$ $43^\circ 7' 48.18''$ $24^\circ 19' 35.22''$	$36^\circ 20' 6.73''$ $34^\circ 1' 41.19''$ $28^\circ 17' 43.37''$	9.9266415	
129. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=4.2893. \end{cases}$	$42^\circ 38' 39.62''$ $22^\circ 11' 35.74''$ $23^\circ 46' 13.46''$	$43^\circ 31' 57.71''$ $22^\circ 29' 33.12''$ $24^\circ 28' 3.70''$	$43^\circ 44' 13.42''$ $22^\circ 26' 30.38''$ $24^\circ 41' 24.64''$	$43^\circ 20' 20.56''$ $22^\circ 39' 55.74''$ $24^\circ 1' 31.91''$	$70^\circ 34' 50.45''$ $13^\circ 16' 3.90''$ $14^\circ 13' 40.88''$	$70^\circ 25' 58.44''$ $13^\circ 8' 48.98''$ $13^\circ 55' 35.62''$	5.314448	
130. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=4.2893. \end{cases}$	$42^\circ 38' 39.62''$ $23^\circ 37' 55.51''$ $22^\circ 14' 22.20''$	$43^\circ 31' 57.71''$ $22^\circ 31' 9.76''$ $24^\circ 20' 44.08''$	$43^\circ 44' 13.42''$ $24^\circ 1' 31.91''$ $24^\circ 41' 24.64''$	$43^\circ 20' 20.56''$ $22^\circ 39' 55.74''$ $22^\circ 26' 30.38''$	$70^\circ 34' 50.45''$ $13^\circ 16' 3.90''$ $14^\circ 13' 40.88''$	$70^\circ 41' 21.10''$ $13^\circ 41' 37.30''$ $13^\circ 0' 50.23''$	5.241143	

[illegible]

See Local File.

2000

EXAMPLES. X.—Continued.

TA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}V, \frac{1}{2}V', \frac{1}{2}V''$	VOLUME.	REMARKS.
141. $\begin{cases} x=4.8206 \\ y=3.5152 \\ z=5.0258 \end{cases}$ $\begin{cases} a=3.4470, \\ b=4.9916, \\ c=4.8206 \end{cases}$	$34^{\circ}17'51.19''$ $22^{\circ}22'20.79''$ $30^{\circ}40'56.99''$	$32^{\circ}59'41.69''$ $28^{\circ}59'17.05''$ $21^{\circ}49'53.13''$	$30^{\circ}19'45.97''$ $20^{\circ}53'1.33''$ $20^{\circ}52'25.74''$	$49^{\circ}18'32.84''$ $34^{\circ}49'43.07''$ $36^{\circ}7'16.98''$	$49^{\circ}18'32.84''$ $34^{\circ}49'43.07''$ $36^{\circ}7'16.98''$	$43^{\circ}51'35.11''$ $24^{\circ}32'5.12''$ $35^{\circ}11'31.67''$	8.596395	A set of ten tetrahedra having the same six lengths for their edges. $V_1 = V_2 = V_3$ $A_1 + A_2 = B_1 + B_3 = 180^{\circ}$
142. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=4.8206 \end{cases}$	$23^{\circ}11'2.51''$ $35^{\circ}12'7.16''$ $29^{\circ}18'11.27''$	$20^{\circ}53'1.33''$ $29^{\circ}38'21.86''$ $20^{\circ}7'46.86''$	$31^{\circ}3'26.84''$ $36^{\circ}7'16.98''$ $22^{\circ}36'43.56''$	$34^{\circ}40'5.98''$ $44^{\circ}12'13.93''$ $32^{\circ}59'41.69''$	$34^{\circ}40'5.98''$ $44^{\circ}12'13.93''$ $32^{\circ}59'41.69''$	$25^{\circ}4'8.33''$ $40^{\circ}6'56.94''$ $32^{\circ}26'19.08''$	8.596395	$B_2 = A_3$ $V_1 = V_2 = V_3$ $A_4 + A_5 = B_4 + B_6 = 180^{\circ}$
143. $\begin{cases} x=4.8206 \\ y=3.4476 \\ z=3.5152 \end{cases}$	$29^{\circ}11'10.80''$ $20^{\circ}32'31.41''$ $20^{\circ}28'51.49''$	$43^{\circ}48'45.10''$ $33^{\circ}30'26.45''$ $34^{\circ}59'23.90''$	$35^{\circ}49'28.73''$ $23^{\circ}23'1.09''$ $31^{\circ}17'39.03''$	$34^{\circ}28'4.69''$ $29^{\circ}31'10.17''$ $22^{\circ}48'13.81''$	$45^{\circ}1'21.50''$ $33^{\circ}13'3.73''$ $34^{\circ}58'13.13''$	$47^{\circ}59'4.02''$ $25^{\circ}4'11.62''$ $24^{\circ}59'10.08''$	8.697423	$B_2 = A_3$ $V_1 = V_2 = V_3$ $A_4 + A_5 = B_4 + B_6 = 180^{\circ}$
144. $\begin{cases} x=4.8206 \\ y=3.4476 \\ z=3.5152 \end{cases}$	$29^{\circ}11'10.80''$ $20^{\circ}7'46.86''$ $20^{\circ}54'6.54''$	$43^{\circ}48'45.10''$ $33^{\circ}30'26.45''$ $35^{\circ}12'7.16''$	$35^{\circ}49'28.73''$ $23^{\circ}23'1.09''$ $31^{\circ}17'39.03''$	$34^{\circ}40'5.98''$ $29^{\circ}31'10.17''$ $22^{\circ}48'13.81''$	$45^{\circ}1'21.50''$ $33^{\circ}13'3.73''$ $34^{\circ}58'13.13''$	$47^{\circ}59'4.02''$ $25^{\circ}4'11.62''$ $24^{\circ}59'10.08''$	8.697421	$B_2 = A_3$ $V_1 = V_2 = V_3$ $A_4 + A_5 = B_4 + B_6 = 180^{\circ}$
145. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=3.4476 \end{cases}$	$21^{\circ}18'10.40''$ $30^{\circ}47'44.97''$ $20^{\circ}28'51.49''$	$22^{\circ}9'57.78''$ $33^{\circ}41'39.78''$ $28^{\circ}45'52.84''$	$35^{\circ}49'28.73''$ $46^{\circ}12'4.72''$ $34^{\circ}5'19.96''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	8.498537	Taking a, b, c, x, y, z to have the meaning of (135) $a^2 + x^2 = 2(c^2 + z^2)$ $3a^2 + 2x^2 = 3b^2 + 2y^2$
146. $\begin{cases} x=4.8206 \\ y=3.4476 \\ z=3.5152 \end{cases}$	$20^{\circ}52'25.74''$ $30^{\circ}47'44.97''$ $20^{\circ}54'6.54''$	$21^{\circ}37'57.50''$ $33^{\circ}30'26.45''$ $28^{\circ}45'52.84''$	$35^{\circ}35'27.01''$ $46^{\circ}12'4.72''$ $34^{\circ}17'51.19''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	8.498537	
147. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=3.4476 \end{cases}$	$17^{\circ}8'33.24''$ $19^{\circ}22'54.45''$ $22^{\circ}49'45.12''$	$25^{\circ}31'33.85''$ $30^{\circ}41'32.71''$ $39^{\circ}48'49.52''$	$36^{\circ}28'42.17''$ $30^{\circ}44'43.67''$ $45^{\circ}15'30.45''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	28.61265	
148. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=3.4476 \end{cases}$	$17^{\circ}11'37.39''$ $19^{\circ}22'54.45''$ $22^{\circ}49'45.12''$	$25^{\circ}31'33.85''$ $30^{\circ}41'32.71''$ $39^{\circ}48'49.52''$	$36^{\circ}28'42.17''$ $30^{\circ}44'43.67''$ $45^{\circ}15'30.45''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	28.62442	In the four tetrahedra, (147) to (150), $a=7.8293, x=4.9316, b+y=a+x, c=a-x$; and (150) is the one of maximum volume under these conditions.
149. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=3.4476 \end{cases}$	$17^{\circ}11'52.27''$ $19^{\circ}22'54.45''$ $22^{\circ}49'45.12''$	$25^{\circ}31'33.85''$ $30^{\circ}41'32.71''$ $39^{\circ}48'49.52''$	$36^{\circ}28'42.17''$ $30^{\circ}44'43.67''$ $45^{\circ}15'30.45''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	28.62443	
150. $\begin{cases} x=4.8206 \\ y=5.0258 \\ z=3.4476 \end{cases}$	$17^{\circ}11'52.27''$ $19^{\circ}22'54.45''$ $22^{\circ}49'45.12''$	$25^{\circ}31'33.85''$ $30^{\circ}41'32.71''$ $39^{\circ}48'49.52''$	$36^{\circ}28'42.17''$ $30^{\circ}44'43.67''$ $45^{\circ}15'30.45''$	$30^{\circ}26'22.19''$ $34^{\circ}36'29.64''$ $23^{\circ}23'1.09''$	$34^{\circ}58'38.50''$ $44^{\circ}58'40.52''$ $33^{\circ}1'33.68''$	$47^{\circ}57'45.17''$ $24^{\circ}30'31.56''$ $25^{\circ}34'9.99''$	28.624435	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}a_5, \frac{1}{2}b_5, \frac{1}{2}c_5$	$\frac{1}{2}a_6, \frac{1}{2}b_6, \frac{1}{2}c_6$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
151. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$	$\begin{cases} 16^{\circ}49'25.87'' \\ 25^{\circ}8'46.85'' \\ 19^{\circ}57'47.85'' \end{cases}$	$\begin{cases} 19^{\circ}54'24.82'' \\ 31^{\circ}18'25.89'' \\ 27^{\circ}48'15.68'' \end{cases}$	$\begin{cases} 38^{\circ}43'46.26'' \\ 42^{\circ}35'53.04'' \\ 33^{\circ}49'51.15'' \end{cases}$	$\begin{cases} 39^{\circ}12'3.07'' \\ 43^{\circ}35'2.42'' \\ 34^{\circ}44.47'' \end{cases}$	$\begin{cases} 23^{\circ}2'57.47'' \\ 37^{\circ}20'42.98'' \\ 27^{\circ}29'42.14'' \end{cases}$	$\begin{cases} 39^{\circ}12'3.07'' \\ 43^{\circ}35'2.42'' \\ 34^{\circ}44.47'' \end{cases}$	$\begin{cases} 23^{\circ}2'57.47'' \\ 37^{\circ}20'42.98'' \\ 27^{\circ}29'42.14'' \end{cases}$	$\begin{cases} 23^{\circ}2'57.47'' \\ 37^{\circ}20'42.98'' \\ 27^{\circ}29'42.14'' \end{cases}$	38.5523	
152. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$	$\begin{cases} 16^{\circ}49'25.87'' \\ 20^{\circ}16'30.92'' \\ 24^{\circ}37'29.66'' \end{cases}$	$\begin{cases} 19^{\circ}54'24.82'' \\ 29^{\circ}44'25.39'' \\ 29^{\circ}1'10.18'' \end{cases}$	$\begin{cases} 35^{\circ}38'4.95'' \\ 27^{\circ}29'42.14'' \\ 33^{\circ}49'51.15'' \end{cases}$	$\begin{cases} 40^{\circ}42'18.90'' \\ 39^{\circ}37'49.63'' \\ 37^{\circ}41'16.24'' \end{cases}$	$\begin{cases} 27^{\circ}27'13.26'' \\ 29^{\circ}28'49.69'' \\ 43^{\circ}23'24.62'' \end{cases}$	$\begin{cases} 41^{\circ}5'26.07'' \\ 39^{\circ}37'49.63'' \\ 37^{\circ}41'16.24'' \end{cases}$	$\begin{cases} 27^{\circ}27'13.26'' \\ 29^{\circ}28'49.69'' \\ 43^{\circ}23'24.62'' \end{cases}$	$\begin{cases} 27^{\circ}27'13.26'' \\ 29^{\circ}28'49.69'' \\ 43^{\circ}23'24.62'' \end{cases}$	38.99001	Set of thirty tetrahedra having all the same six lengths for their edges. (151) to (156) is Group I., in which 9.4106, 4.8370 are opposite. (157) to (162) is Group II., in which 9.4106, 6.1874 are opposite. (163) to (168) is Group III., in which 9.4106, 7.5278 are opposite. (169) to (174) is Group IV., in which 9.4106, 8.0746 are opposite; and lastly, Group V., in which edges 9.4106, 8.5602 are opposite. The average volume is considerably the least in this group; indeed, 5 out of the 6 least are in this group.
153. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{cases}$	$\begin{cases} 17^{\circ}7'4.08'' \\ 27^{\circ}48'15.68'' \\ 19^{\circ}57'47.85'' \end{cases}$	$\begin{cases} 18^{\circ}21'48.60'' \\ 31^{\circ}18'25.89'' \\ 25^{\circ}8'46.85'' \end{cases}$	$\begin{cases} 38^{\circ}43'46.26'' \\ 46^{\circ}41'6.12'' \\ 30^{\circ}33'22.03'' \end{cases}$	$\begin{cases} 37^{\circ}2'57.47'' \\ 42^{\circ}19'33.89'' \\ 24^{\circ}57'5.28'' \end{cases}$	$\begin{cases} 27^{\circ}27'13.26'' \\ 29^{\circ}28'49.69'' \\ 43^{\circ}23'24.62'' \end{cases}$	$\begin{cases} 38^{\circ}50'33.15'' \\ 45^{\circ}14'57.84'' \\ 29^{\circ}42'21.74'' \end{cases}$	$\begin{cases} 21^{\circ}0'2.88'' \\ 50^{\circ}31'14.11'' \\ 24^{\circ}52'50.62'' \end{cases}$	$\begin{cases} 21^{\circ}0'2.88'' \\ 50^{\circ}31'14.11'' \\ 24^{\circ}52'50.62'' \end{cases}$	35.81829	
154. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$	$\begin{cases} 17^{\circ}7'4.08'' \\ 20^{\circ}29'44.11'' \\ 26^{\circ}34'18.74'' \end{cases}$	$\begin{cases} 18^{\circ}21'48.60'' \\ 29^{\circ}041.86'' \\ 26^{\circ}28'18.40'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 24^{\circ}57'5.28'' \\ 30^{\circ}33'22.03'' \end{cases}$	$\begin{cases} 43^{\circ}1'57.40'' \\ 42^{\circ}19'33.89'' \\ 46^{\circ}41'6.12'' \end{cases}$	$\begin{cases} 22^{\circ}16'40.71'' \\ 27^{\circ}26'35.89'' \\ 46^{\circ}52'52.44'' \end{cases}$	$\begin{cases} 42^{\circ}51'44.14'' \\ 42^{\circ}11'54.78'' \\ 34^{\circ}43'10.94'' \end{cases}$	$\begin{cases} 22^{\circ}16'40.71'' \\ 27^{\circ}26'35.89'' \\ 46^{\circ}52'52.44'' \end{cases}$	$\begin{cases} 22^{\circ}16'40.71'' \\ 27^{\circ}26'35.89'' \\ 46^{\circ}52'52.44'' \end{cases}$	37.21727	
155. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$	$\begin{cases} 16^{\circ}49'58.84'' \\ 26^{\circ}28'18.40'' \\ 26^{\circ}34'18.74'' \end{cases}$	$\begin{cases} 17^{\circ}57'6.85'' \\ 29^{\circ}041.86'' \\ 20^{\circ}29'44.11'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 32^{\circ}56'14.05'' \\ 22^{\circ}34'53.39'' \end{cases}$	$\begin{cases} 43^{\circ}1'57.40'' \\ 50^{\circ}35'7.77'' \\ 39^{\circ}6'39.10'' \end{cases}$	$\begin{cases} 21^{\circ}14'48.00'' \\ 38^{\circ}15'37.49'' \\ 38^{\circ}34'57.26'' \end{cases}$	$\begin{cases} 41^{\circ}15'59.80'' \\ 51^{\circ}8'26.42'' \\ 24^{\circ}32'17.15'' \end{cases}$	$\begin{cases} 21^{\circ}14'48.00'' \\ 38^{\circ}15'37.49'' \\ 38^{\circ}34'57.26'' \end{cases}$	$\begin{cases} 21^{\circ}14'48.00'' \\ 38^{\circ}15'37.49'' \\ 38^{\circ}34'57.26'' \end{cases}$	35.83271	
156. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$	$\begin{cases} 16^{\circ}49'58.84'' \\ 29^{\circ}1'10.18'' \\ 24^{\circ}37'29.66'' \end{cases}$	$\begin{cases} 17^{\circ}57'6.85'' \\ 29^{\circ}44'25.39'' \\ 20^{\circ}16'30.92'' \end{cases}$	$\begin{cases} 35^{\circ}38'4.95'' \\ 39^{\circ}6'39.10'' \\ 22^{\circ}34'53.39'' \end{cases}$	$\begin{cases} 40^{\circ}42'18.90'' \\ 50^{\circ}35'7.77'' \\ 39^{\circ}6'39.10'' \end{cases}$	$\begin{cases} 20^{\circ}23'8.27'' \\ 45^{\circ}57'4.25'' \\ 31^{\circ}35'24.39'' \end{cases}$	$\begin{cases} 37^{\circ}36'20.76'' \\ 53^{\circ}11'59.38'' \\ 23^{\circ}11'29.52'' \end{cases}$	$\begin{cases} 20^{\circ}23'8.27'' \\ 45^{\circ}57'4.25'' \\ 31^{\circ}35'24.39'' \end{cases}$	$\begin{cases} 20^{\circ}23'8.27'' \\ 45^{\circ}57'4.25'' \\ 31^{\circ}35'24.39'' \end{cases}$	34.86761	
157. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$	$\begin{cases} 21^{\circ}46'39.73'' \\ 25^{\circ}8'46.85'' \\ 15^{\circ}22'57.43'' \end{cases}$	$\begin{cases} 27^{\circ}29'42.14'' \\ 32^{\circ}25'44.16'' \\ 27^{\circ}48'15.68'' \end{cases}$	$\begin{cases} 42^{\circ}11'18.41'' \\ 42^{\circ}35'53.04'' \\ 32^{\circ}1'21.46'' \end{cases}$	$\begin{cases} 37^{\circ}2'57.47'' \\ 36^{\circ}12'7.81'' \\ 19^{\circ}54'24.82'' \end{cases}$	$\begin{cases} 31^{\circ}43'55.28'' \\ 43^{\circ}41'28.89'' \\ 20^{\circ}48'57.02'' \end{cases}$	$\begin{cases} 43^{\circ}2'14.82'' \\ 40^{\circ}42'40.59'' \\ 32^{\circ}10'9.35'' \end{cases}$	$\begin{cases} 31^{\circ}43'55.28'' \\ 43^{\circ}41'28.89'' \\ 20^{\circ}48'57.02'' \end{cases}$	$\begin{cases} 31^{\circ}43'55.28'' \\ 43^{\circ}41'28.89'' \\ 20^{\circ}48'57.02'' \end{cases}$	38.16771	
158. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$	$\begin{cases} 21^{\circ}46'39.73'' \\ 15^{\circ}27'54.53'' \\ 24^{\circ}37'29.66'' \end{cases}$	$\begin{cases} 27^{\circ}29'42.14'' \\ 29^{\circ}44'25.39'' \\ 29^{\circ}32'53.45'' \end{cases}$	$\begin{cases} 35^{\circ}38'4.95'' \\ 19^{\circ}54'24.82'' \\ 32^{\circ}1'21.46'' \end{cases}$	$\begin{cases} 44^{\circ}59'12.02'' \\ 36^{\circ}12'7.81'' \\ 42^{\circ}35'53.04'' \end{cases}$	$\begin{cases} 32^{\circ}28'40.22'' \\ 21^{\circ}5'38.60'' \\ 42^{\circ}29'2.81'' \end{cases}$	$\begin{cases} 45^{\circ}44'58.47'' \\ 36^{\circ}10'16.71'' \\ 35^{\circ}49'12.37'' \end{cases}$	$\begin{cases} 32^{\circ}28'40.22'' \\ 21^{\circ}5'38.60'' \\ 42^{\circ}29'2.81'' \end{cases}$	$\begin{cases} 32^{\circ}28'40.22'' \\ 21^{\circ}5'38.60'' \\ 42^{\circ}29'2.81'' \end{cases}$	38.24435	
159. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=4.8370, \end{cases}$	$\begin{cases} 22^{\circ}34'53.39'' \\ 29^{\circ}32'53.45'' \\ 24^{\circ}37'29.66'' \end{cases}$	$\begin{cases} 23^{\circ}16'55.53'' \\ 29^{\circ}44'25.39'' \\ 15^{\circ}27'54.53'' \end{cases}$	$\begin{cases} 35^{\circ}38'4.95'' \\ 35^{\circ}41'13.44'' \\ 16^{\circ}49'58.84'' \end{cases}$	$\begin{cases} 44^{\circ}59'12.02'' \\ 50^{\circ}35'7.77'' \\ 31^{\circ}1'51.03'' \end{cases}$	$\begin{cases} 27^{\circ}39'39.12'' \\ 42^{\circ}9'3.20'' \\ 30^{\circ}44'1.05'' \end{cases}$	$\begin{cases} 41^{\circ}58'43.52'' \\ 51^{\circ}20'27.93'' \\ 17^{\circ}47'57.02'' \end{cases}$	$\begin{cases} 27^{\circ}39'39.12'' \\ 42^{\circ}9'3.20'' \\ 30^{\circ}44'1.05'' \end{cases}$	$\begin{cases} 27^{\circ}39'39.12'' \\ 42^{\circ}9'3.20'' \\ 30^{\circ}44'1.05'' \end{cases}$	34.71445	
160. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=4.8370, \end{cases}$	$\begin{cases} 22^{\circ}34'53.39'' \\ 26^{\circ}18'8.81'' \\ 26^{\circ}34'18.74'' \end{cases}$	$\begin{cases} 23^{\circ}16'55.53'' \\ 29^{\circ}041.86'' \\ 15^{\circ}20'53.15'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 31^{\circ}1'51.03'' \\ 16^{\circ}49'58.84'' \end{cases}$	$\begin{cases} 48^{\circ}20'58.04'' \\ 50^{\circ}35'7.77'' \\ 35^{\circ}41'13.44'' \end{cases}$	$\begin{cases} 28^{\circ}47'20.53'' \\ 35^{\circ}31'2.79'' \\ 36^{\circ}7'58.78'' \end{cases}$	$\begin{cases} 46^{\circ}41'19.92'' \\ 49^{\circ}47'7.25'' \\ 18^{\circ}12'0.79'' \end{cases}$	$\begin{cases} 28^{\circ}47'20.53'' \\ 35^{\circ}31'2.79'' \\ 36^{\circ}7'58.78'' \end{cases}$	$\begin{cases} 28^{\circ}47'20.53'' \\ 35^{\circ}31'2.79'' \\ 36^{\circ}7'58.78'' \end{cases}$	34.84775	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
71. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 4' 19.49'' \\ 20^\circ 49' 36.58'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 33^\circ 10' 59.37'' \\ 34^\circ 14' 11.92'' \\ 22^\circ 25' 39.39'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 34^\circ 11' 57.06'' \\ 24^\circ 17' 59.87'' \end{cases}$	$\begin{cases} 46^\circ 44' 44.03'' \\ 43^\circ 37' 40.64'' \\ 22^\circ 37' 3.57'' \end{cases}$	$\begin{cases} 48^\circ 49' 59.22'' \\ 41^\circ 19' 16.21'' \\ 24^\circ 22' 49.11'' \end{cases}$	$\begin{cases} 38^\circ 48' 19.55'' \\ 34^\circ 22' 45.17'' \\ 22^\circ 24' 48.43'' \end{cases}$	13.77206	
72. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 4' 19.49'' \\ 14^\circ 18' 6.32'' \\ 19^\circ 59' 26.14'' \end{cases}$	$\begin{cases} 33^\circ 10' 59.37'' \\ 32^\circ 36' 52.49'' \\ 20^\circ 51' 11.06'' \end{cases}$	$\begin{cases} 37^\circ 23' 41.37'' \\ 22^\circ 37' 3.57'' \\ 24^\circ 17' 59.87'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 43^\circ 37' 40.64'' \\ 34^\circ 11' 57.06'' \end{cases}$	$\begin{cases} 56^\circ 13' 19.51'' \\ 39^\circ 20' 13.50'' \\ 22^\circ 57' 49.36'' \end{cases}$	$\begin{cases} 40^\circ 48' 28.45'' \\ 21^\circ 25' 25.05'' \\ 32^\circ 56' 27.75'' \end{cases}$	12.84365	
73. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 6' 5.66'' \\ 22^\circ 25' 39.39'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 31^\circ 26' 51.14'' \\ 34^\circ 14' 11.92'' \\ 20^\circ 49' 36.58'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 36^\circ 44' 30.25'' \\ 22^\circ 31' 9.65'' \end{cases}$	$\begin{cases} 46^\circ 44' 44.03'' \\ 45^\circ 22' 44.69'' \\ 21^\circ 48' 38.61'' \end{cases}$	$\begin{cases} 47^\circ 7' 58.93'' \\ 43^\circ 42' 58.08'' \\ 22^\circ 46' 12.81'' \end{cases}$	$\begin{cases} 36^\circ 27' 59.09'' \\ 37^\circ 37' 46.06'' \\ 21^\circ 47' 8.56'' \end{cases}$	13.47945	
74. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 6' 5.66'' \\ 13^\circ 34' 58.03'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 31^\circ 26' 51.14'' \\ 32^\circ 7' 34.19'' \\ 18^\circ 21' 32.66'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 21^\circ 48' 38.61'' \\ 22^\circ 31' 9.65'' \end{cases}$	$\begin{cases} 58^\circ 3' 29.31'' \\ 45^\circ 22' 44.69'' \\ 36^\circ 44' 30.25'' \end{cases}$	$\begin{cases} 58^\circ 32' 26.51'' \\ 41^\circ 15' 54.61'' \\ 20^\circ 34' 45.94'' \end{cases}$	$\begin{cases} 39^\circ 14' 51.79'' \\ 19^\circ 57' 32.53'' \\ 35^\circ 51' 56.23'' \end{cases}$	12.03492	(71) to (84) are also tetrahedra with the same six lengths for their edges. The six tetrahedra in which the two longest edges are opposite each other are all impossible.
75. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 20^\circ 53' 38.77'' \\ 20^\circ 51' 11.06'' \\ 19^\circ 59' 26.14'' \end{cases}$	$\begin{cases} 28^\circ 30' 37.55'' \\ 32^\circ 36' 52.49'' \\ 14^\circ 18' 6.32'' \end{cases}$	$\begin{cases} 37^\circ 23' 41.37'' \\ 32^\circ 18' 35.14'' \\ 15^\circ 32' 49.76'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 53^\circ 33' 31.47'' \\ 29^\circ 10' 47.31'' \end{cases}$	$\begin{cases} 51^\circ 28' 55.49'' \\ 49^\circ 13' 2.84'' \\ 15^\circ 43' 5.90'' \end{cases}$	$\begin{cases} 33^\circ 1' 24.98'' \\ 32^\circ 55' 14.21'' \\ 30^\circ 53' 33.48'' \end{cases}$	11.86436	
76. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 20^\circ 53' 38.77'' \\ 18^\circ 21' 32.66'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 28^\circ 30' 37.55'' \\ 32^\circ 7' 34.19'' \\ 13^\circ 34' 58.03'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 29^\circ 10' 47.31'' \\ 15^\circ 32' 49.76'' \end{cases}$	$\begin{cases} 58^\circ 3' 29.31'' \\ 53^\circ 33' 31.47'' \\ 32^\circ 18' 35.14'' \end{cases}$	$\begin{cases} 55^\circ 39' 49.09'' \\ 48^\circ 44' 29.87'' \\ 15^\circ 5' 6.60'' \end{cases}$	$\begin{cases} 33^\circ 42' 51.86'' \\ 27^\circ 58' 17.77'' \\ 34^\circ 48' 7.14'' \end{cases}$	11.34094	
77. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 28' 7.17'' \\ 14^\circ 18' 6.32'' \\ 19^\circ 40' 57.73'' \end{cases}$	$\begin{cases} 34^\circ 11' 57.06'' \\ 32^\circ 43' 41.30'' \\ 20^\circ 51' 11.06'' \end{cases}$	$\begin{cases} 37^\circ 35' 20.97'' \\ 22^\circ 37' 3.57'' \\ 24^\circ 16' 36.86'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 43^\circ 15' 15.97'' \\ 33^\circ 10' 59.37'' \end{cases}$	$\begin{cases} 56^\circ 38' 17.14'' \\ 38^\circ 26' 2.57'' \\ 22^\circ 42' 35.79'' \end{cases}$	$\begin{cases} 42^\circ 14' 54.68'' \\ 21^\circ 12' 50.88'' \\ 31^\circ 40' 45.14'' \end{cases}$	12.7562	
78. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 22^\circ 31' 9.65'' \\ 22^\circ 23' 55.02'' \\ 15^\circ 5' 20.86'' \end{cases}$	$\begin{cases} 32^\circ 1' 22.37'' \\ 34^\circ 14' 11.92'' \\ 20^\circ 51' 11.06'' \end{cases}$	$\begin{cases} 40^\circ 40' 27.22'' \\ 36^\circ 17' 43.87'' \\ 22^\circ 6' 5.66'' \end{cases}$	$\begin{cases} 47^\circ 10' 17.32'' \\ 45^\circ 22' 44.69'' \\ 21^\circ 40' 53.76'' \end{cases}$	$\begin{cases} 47^\circ 45' 45.65'' \\ 43^\circ 14' 56.18'' \\ 22^\circ 17' 30.93'' \end{cases}$	$\begin{cases} 37^\circ 22' 19.32'' \\ 36^\circ 56' 27.33'' \\ 21^\circ 38' 0.51'' \end{cases}$	13.39033	
79. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 31' 9.65'' \\ 13^\circ 22' 3.20'' \\ 21^\circ 16' 55.00'' \end{cases}$	$\begin{cases} 32^\circ 1' 22.37'' \\ 32^\circ 7' 34.19'' \\ 17^\circ 44' 30.19'' \end{cases}$	$\begin{cases} 36^\circ 35' 30.81'' \\ 21^\circ 40' 53.76'' \\ 22^\circ 6' 5.66'' \end{cases}$	$\begin{cases} 58^\circ 53' 26.61'' \\ 45^\circ 22' 44.69'' \\ 36^\circ 17' 43.87'' \end{cases}$	$\begin{cases} 59^\circ 29' 8.08'' \\ 40^\circ 42' 2.21'' \\ 19^\circ 47' 31.02'' \end{cases}$	$\begin{cases} 40^\circ 22' 54.14'' \\ 19^\circ 26' 3.19'' \\ 35^\circ 19' 41.99'' \end{cases}$	11.7694	
80. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\begin{cases} 21^\circ 28' 4.14'' \\ 20^\circ 51' 11.06'' \\ 19^\circ 40' 57.73'' \end{cases}$	$\begin{cases} 29^\circ 10' 47.31'' \\ 32^\circ 43' 41.30'' \\ 14^\circ 18' 6.32'' \end{cases}$	$\begin{cases} 37^\circ 35' 20.97'' \\ 32^\circ 18' 35.14'' \\ 15^\circ 40' 18.85'' \end{cases}$	$\begin{cases} 54^\circ 50' 42.62'' \\ 52^\circ 18' 35.14'' \\ 28^\circ 30' 37.55'' \end{cases}$	$\begin{cases} 51^\circ 47' 4.39'' \\ 48^\circ 10' 54.20'' \\ 15^\circ 46' 7.56'' \end{cases}$	$\begin{cases} 34^\circ 13' 46.68'' \\ 32^\circ 38' 55.92'' \\ 30^\circ 0' 14.77'' \end{cases}$	11.9202	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
81. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 24°16'36.86" 13°34'58.03" 19°40'57.73"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 36°44'30.25" 32°43'41.30" 18°21'32.60"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 37°35'20.97" 21°48'38.61" 22°28'7.17"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 58°3'29.31" 43°15'15.97" 31°20'51.14"	60°53'22.78" 35°27'11.69" 19°11'54.74"	47°34'3.05" 18°40'25.74" 28°43'0.00"	11.47017	
82. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7557, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 24°17'59.87" 13°22'3.20" 19°59'26.14"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 36°17'43.87" 32°30'52.49" 17°44'30.19"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 37°23'41.37" 21°40'53.76" 22°4'19.49"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 58°53'26.61" 43°37'40.64" 32°1'22.37"	61°28'32.90" 35°46'15.30" 18°39'53.35"	47°17'4.47" 18°21'26.04" 29°19'2.54"	11.29084	
83. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7757, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 24°17'59.87" 20°36'26.56" 15°5'20.86"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 36°17'43.87" 34°14'11.92" 20°14'28.81"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 40°40'27.22" 32°1'22.37" 22°4'19.49"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 49°9'4.63" 43°37'40.64" 21°40'53.76"	52°34'24.73" 38°29'56.95" 21°25'19.56"	44°2'47.21" 30°41'46.11" 21°1'31.02"	12.97171	See previous page.
84. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 23°49'31.44" 18°21'32.66" 19°40'57.73"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 32°18'35.14" 32°43'41.30" 15°34'58.03"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 37°35'20.97" 29°10'47.31" 16°3'50.02"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 58°3'29.31" 50°6'38.54" 28°30'37.55"	57°17'51.54" 42°37'59.24" 15°0'24.06"	40°54'43.74" 26°36'4.61" 29°4'41.10"	11.39934	
85. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 43°15'15.97" 20°14'28.81" 14°18'6.32"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 53°33'31.47" 20°51'11.06" 20°36'26.56"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 54°50'42.62" 20°53'38.77" 22°28'7.17"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 49°9'4.63" 24°16'36.86" 15°32'49.76"	Impossible	Imp.	Imp.	
86. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 43°15'15.97" 20°14'28.81" 23°57'19.97"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 42°8'30.59" 22°26'49.69" 20°36'26.56"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 43°35'52.96" 21°57'56.74" 22°28'7.17"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 48°9'4.63" 24°16'36.86" 25°53'32.67"	80°3'52.52" 7°27'37.12" 6°57'15.17"	79°21'27.65" 6°49'42.21" 7°49'43.59"	5.223158	These six, (86) to (91), are those, out of the thirty which have the same six lengths for their edges, in which the two longest edges are opposite. They are all "ill-conditioned," and their volume is much less than in any similar group of six.
87. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.7557, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 43°15'15.97" 23°59'35.70" 20°25'47.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 42°8'30.59" 22°23'55.02" 20°32'26.27"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 47°10'17.32" 25°53'32.67" 22°28'7.17"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 48°9'4.63" 24°16'36.86" 21°57'56.74"	78°53'27.10" 8°14'42.14" 7°46'47.28"	78°11'3.83" 8°40'48.46" 7°36'57.44"	5.81898	
88. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.7557, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 43°37'40.64" 20°36'26.56" 23°57'19.97"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 41°43'28.77" 22°26'47.07" 20°14'28.81"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 43°35'52.96" 22°20'40.15" 22°4'19.49"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 48°9'4.63" 24°17'59.87" 25°50'51.12"	80°10'10.97" 7°23'17.26" 6°47'2.96"	79°28'36.49" 6°51'5.74" 7°44'14.56"	5.167364	
89. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 43°37'40.64" 23°59'18.49" 20°49'36.58"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 41°48'28.73" 22°25'39.39" 20°29'50.39"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 46°44'44.03" 25°50'51.12" 22°4'19.49"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 45°30'51.12" 24°17'59.87" 22°20'40.15"	78°51'15.86" 8°15'59.98" 7°40'25.27"	78°10'50.57" 8°40'30.20" 7°44'12.92"	5.825749	
90. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=5.3856, \end{cases}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 40°21'9.94" 20°52'26.27" 20°25'47.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 45°22'44.60" 22°23'55.02" 23°59'35.70"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 47°10'17.32" 22°31'9.55" 25°36'58.48"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 45°7'58.03" 22°55'51.58" 22°6'5.66"	79°57'50.92" 7°46'59.86"	79°24'51.77" 7°2'51.96" 6°55'19.33"	5.2427945	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
91. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=5.3856, \end{cases}$ $\begin{cases} x=6.8205 \\ y=4.7557 \\ z=4.8205 \end{cases}$	$40^\circ 27' 9.94''$ $20^\circ 26' 50.39''$ $20^\circ 49' 36.58''$	$45^\circ 22' 44.69''$ $22^\circ 25' 39.39''$ $23^\circ 59' 18.49''$	$46^\circ 44' 44.03''$ $22^\circ 6' 5.66''$ $25^\circ 36' 58.48''$	$45^\circ 30' 51.12''$ $23^\circ 55' 51.58''$ $22^\circ 31' 9.65''$	$70^\circ 30' 21.87''$ $7^\circ 27' 45.82''$ $7^\circ 52' 12.49''$	$79^\circ 17' 30.88''$ $7^\circ 1' 9.61''$ $7^\circ 6' 49.33''$	5.305218	See previous page.
92. $\begin{cases} a=7.3871, \\ b=5.9635, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.9870 \\ y=5.9634 \\ z=5.7634 \end{cases}$	$35^\circ 10' 9.55''$ $25^\circ 37' 31.80''$ $24^\circ 52' 47.74''$	$36^\circ 33' 34.10''$ $26^\circ 5' 8.05''$ $26^\circ 51' 23.75''$	$39^\circ 2' 4.21''$ $27^\circ 22' 44.36''$ $28^\circ 5' 8.66''$	$37^\circ 31' 4.45''$ $26^\circ 44' 1.79''$ $26^\circ 5' 41.45''$	$49^\circ 51' 29.66''$ $28^\circ 32' 38.29''$ $28^\circ 24' 19.82''$	$48^\circ 16' 23.66''$ $27^\circ 40' 55.35''$ $26^\circ 49' 13.82''$	26.76496	
93. $\begin{cases} a=7.1471, \\ b=5.9669, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.7470 \\ y=5.9600 \\ z=5.7634 \end{cases}$	$33^\circ 46' 57.23''$ $26^\circ 17' 5.52''$ $25^\circ 35' 23.03''$	$35^\circ 7' 27.48''$ $26^\circ 53' 1.12''$ $27^\circ 30' 10.53''$	$37^\circ 31' 35.85''$ $28^\circ 7' 14.44''$ $28^\circ 48' 9.39''$	$36^\circ 6' 43.95''$ $27^\circ 24' 53.38''$ $26^\circ 45' 18.08''$	$46^\circ 37' 29.15''$ $29^\circ 29' 35.48''$ $30^\circ 22' 39.48''$	$44^\circ 58' 17.27''$ $29^\circ 36' 31.07''$ $28^\circ 43' 27.60''$	27.04064	(92) to (96) exemplify the process of finding the tetrahedron of maximum volume when $a-x$, $b-y$, c , and z are given.
94. $\begin{cases} a=7.0871, \\ b=5.9769, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.6870 \\ y=5.9500 \\ z=5.7634 \end{cases}$	$33^\circ 24' 43.34''$ $26^\circ 24' 9.16''$ $25^\circ 45' 7.03''$	$34^\circ 48' 5.32''$ $27^\circ 7' 42.21''$ $27^\circ 48' 9.75''$	$37^\circ 7' 10.76''$ $28^\circ 15' 20.60''$ $28^\circ 57' 40.82''$	$35^\circ 47' 41.09''$ $26^\circ 56' 33.99''$ $26^\circ 57' 29.12''$	$45^\circ 52' 4.32''$ $30^\circ 58' 33.44''$ $30^\circ 49' 22.42''$	$44^\circ 12' 24.45''$ $30^\circ 0' 31.53''$ $29^\circ 9' 42.55''$	27.11766	
95. $\begin{cases} a=7.0841, \\ b=5.9807, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.6840 \\ y=5.9462 \\ z=5.7634 \end{cases}$	$33^\circ 23' 1.17''$ $26^\circ 23' 32.04''$ $25^\circ 45' 19.88''$	$34^\circ 47' 49.00''$ $27^\circ 9' 27.60''$ $27^\circ 49' 4.82''$	$37^\circ 5' 12.52''$ $28^\circ 14' 41.88''$ $28^\circ 57' 44.80''$	$35^\circ 47' 23.14''$ $27^\circ 39' 14.03''$ $26^\circ 57' 29.12''$	$45^\circ 49' 45.62''$ $30^\circ 0' 48.71''$ $30^\circ 50' 36.16''$	$44^\circ 10' 45.62''$ $30^\circ 0' 52.12''$ $29^\circ 11' 4.12''$	27.117785	
96. $\begin{cases} a=7.084115, \\ b=5.980813, \\ c=6.1635, \end{cases}$ $\begin{cases} x=6.684015 \\ y=5.946087 \\ z=5.7634 \end{cases}$	$33^\circ 23' 0.26''$ $26^\circ 23' 29.90''$ $25^\circ 45' 19.18''$	$34^\circ 47' 50.76''$ $27^\circ 9' 29.52''$ $27^\circ 49' 5.29''$	$37^\circ 5' 11.30''$ $28^\circ 14' 39.56''$ $28^\circ 57' 43.80''$	$35^\circ 47' 24.81''$ $27^\circ 39' 15.94''$ $26^\circ 57' 29.68''$	$45^\circ 49' 46.14''$ $30^\circ 0' 50.02''$ $30^\circ 50' 36.16''$	$44^\circ 10' 13.86''$ $30^\circ 0' 50.02''$ $29^\circ 11' 3.88''$	27.117785	
97. $\begin{cases} a=4.886136, \\ b=3.967768, \\ c=4.3257, \end{cases}$ $\begin{cases} x=3.775036 \\ y=3.572532 \\ z=3.2146 \end{cases}$	$26^\circ 58' 18.90''$ $22^\circ 33' 53.45''$ $20^\circ 28' 36.21''$	$33^\circ 40' 27.35''$ $26^\circ 59' 59.41''$ $29^\circ 33' 10.32''$	$42^\circ 31' 24.38''$ $30^\circ 25' 32.63''$ $33^\circ 56' 15.88''$	$37^\circ 52' 56.23''$ $29^\circ 5' 25.22''$ $25^\circ 54' 0.02''$	$48^\circ 26' 47.83''$ $30^\circ 14' 47.56''$ $33^\circ 41' 35.39''$	$43^\circ 33' 12.17''$ $30^\circ 14' 47.56''$ $26^\circ 47' 59.73''$	6.445679	$A-X=C-Z$, $B+Y=C+Z$, $A+X=180^\circ$, $B-Y=0^\circ$, a similar case of maximum volume.
98. $\begin{cases} a=5.3147, \\ b=4.0257, \\ c=4.3257, \end{cases}$ $\begin{cases} x=4.2036 \\ y=3.2146 \\ z=3.2146 \end{cases}$	$30^\circ 9' 27.67''$ $20^\circ 35' 27.40''$ $18^\circ 33' 42.30''$	$38^\circ 35' 14.65''$ $24^\circ 32' 54.36''$ $27^\circ 4' 4.68''$	$46^\circ 53' 23.34''$ $27^\circ 18' 18.99''$ $31^\circ 41' 24.26''$	$42^\circ 20' 27.92''$ $28^\circ 9' 8.07''$ $24^\circ 6' 26.45''$	$58^\circ 15' 33.72''$ $24^\circ 11' 41.63''$ $26^\circ 38' 44.13''$	$52^\circ 39' 1.48''$ $23^\circ 29' 14.39''$ $21^\circ 2' 11.89''$	5.912593	$A-X=C-Z$, $B+Y=C+Z$
99. $\begin{cases} a=11.99045, \\ b=12.0002, \\ c=15.01087, \end{cases}$ $\begin{cases} x=10.96695 \\ y=9.7828 \\ z=10.70947 \end{cases}$	$23^\circ 7' 58.61''$ $20^\circ 17' 26.61''$ $26^\circ 30' 47.00''$	$32^\circ 15' 47.00''$ $31^\circ 46' 0.51''$ $43^\circ 16' 16.71''$	$31^\circ 43' 12.49''$ $26^\circ 49' 11.31''$ $40^\circ 44' 41.19''$	$26^\circ 26' 16.68''$ $26^\circ 7' 20.20''$ $30^\circ 55' 1.69''$	$31^\circ 35' 16.82''$ $31^\circ 7' 10.35''$ $49^\circ 19' 19.78''$	$31^\circ 35' 16.82''$ $26^\circ 43' 47.56''$ $40^\circ 19' 19.78''$	166.9554	Tetrahedron of max. vol. when $a+x$, $b+y$, and $c-z$ are given.
100. $\begin{cases} a=5.966907, \\ b=6.028293, \\ c=7.5063, \end{cases}$ $\begin{cases} x=5.4836 \\ y=4.8914 \\ z=4.3556 \end{cases}$	$23^\circ 6' 38.24''$ $20^\circ 17' 45.64''$ $21^\circ 17' 22.23''$	$36^\circ 14' 24.17''$ $34^\circ 43' 53.23''$ $43^\circ 26' 9.60''$	$33^\circ 58' 44.54''$ $29^\circ 8' 23.94''$ $40^\circ 37' 17.00''$	$26^\circ 16' 4.76''$ $26^\circ 16' 4.76''$ $24^\circ 37' 11.89''$	$34^\circ 43' 13.30''$ $34^\circ 43' 13.30''$ $46^\circ 38' 17.92''$	$36^\circ 13' 46.82''$ $29^\circ 36' 59.18''$ $31^\circ 39' 31.09''$	18.889605	$a_4=b_4$, $A=B$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
111. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$	$\begin{cases} 22^\circ 39' 55.74'' \\ 23^\circ 51' 55.51'' \\ 20^\circ 35' 48.29'' \end{cases}$	$\begin{cases} 20^\circ 24' 59.22'' \\ 34^\circ 22' 51.03'' \\ 24^\circ 20' 44.08'' \end{cases}$	$\begin{cases} 35^\circ 1' 20.66'' \\ 31^\circ 37' 31.51'' \\ 24^\circ 41' 24.04'' \end{cases}$	$\begin{cases} 42^\circ 1' 20.41'' \\ 42^\circ 38' 39.62'' \\ 28^\circ 57' 29.27'' \end{cases}$	$\begin{cases} 42^\circ 57' 30.25'' \\ 44^\circ 4' 51.88'' \\ 26^\circ 49' 42.42'' \end{cases}$	$\begin{cases} 33^\circ 17' 4.01'' \\ 35^\circ 41' 10.06'' \\ 29^\circ 5' 23.85'' \end{cases}$	10.14892	
112. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$	$\begin{cases} 22^\circ 39' 55.74'' \\ 22^\circ 11' 35.74'' \\ 21^\circ 45' 47.46'' \end{cases}$	$\begin{cases} 20^\circ 24' 59.22'' \\ 33^\circ 48' 59.21'' \\ 24^\circ 28' 3.70'' \end{cases}$	$\begin{cases} 34^\circ 25' 13.33'' \\ 28^\circ 57' 29.27'' \\ 24^\circ 41' 24.04'' \end{cases}$	$\begin{cases} 43^\circ 20' 56'' \\ 42^\circ 38' 39.62'' \\ 31^\circ 37' 31.51'' \end{cases}$	$\begin{cases} 44^\circ 19' 28.80'' \\ 43^\circ 11' 50.97'' \\ 27^\circ 13' 45.53'' \end{cases}$	$\begin{cases} 33^\circ 43' 2.81'' \\ 33^\circ 38' 1.83'' \\ 31^\circ 42' 19.75'' \end{cases}$	10.17192	
113. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$	$\begin{cases} 22^\circ 42' 40.72'' \\ 24^\circ 20' 44.08'' \\ 20^\circ 35' 48.29'' \end{cases}$	$\begin{cases} 29^\circ 0' 25.82'' \\ 34^\circ 22' 51.03'' \\ 23^\circ 27' 55.51'' \end{cases}$	$\begin{cases} 35^\circ 1' 20.66'' \\ 32^\circ 25' 47.57'' \\ 24^\circ 0' 35.31'' \end{cases}$	$\begin{cases} 42^\circ 1' 20.41'' \\ 43^\circ 16' 43.97'' \\ 28^\circ 33' 40.61'' \end{cases}$	$\begin{cases} 42^\circ 33' 55.17'' \\ 44^\circ 52' 20.02'' \\ 25^\circ 59' 58.65'' \end{cases}$	$\begin{cases} 32^\circ 45' 3.72'' \\ 36^\circ 49' 51.66'' \\ 28^\circ 38' 34.07'' \end{cases}$	10.06527	
114. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$	$\begin{cases} 22^\circ 42' 40.72'' \\ 22^\circ 13' 23.52'' \\ 22^\circ 17' 0.28'' \end{cases}$	$\begin{cases} 29^\circ 0' 25.82'' \\ 33^\circ 33' 50.81'' \\ 23^\circ 40' 53.71'' \end{cases}$	$\begin{cases} 34^\circ 9' 8.91'' \\ 28^\circ 33' 46.61'' \\ 24^\circ 0' 35.31'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 43^\circ 16' 43.97'' \\ 32^\circ 25' 47.57'' \end{cases}$	$\begin{cases} 44^\circ 41' 49.10'' \\ 43^\circ 34' 11.58'' \\ 26^\circ 33' 54.34'' \end{cases}$	$\begin{cases} 33^\circ 25' 12.21'' \\ 32^\circ 20' 2.08'' \\ 32^\circ 27' 50.67'' \end{cases}$	10.10116	
115. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$\begin{cases} 22^\circ 45' 46.37'' \\ 23^\circ 46' 53.71'' \\ 22^\circ 17' 0.28'' \end{cases}$	$\begin{cases} 28^\circ 16' 10.80'' \\ 33^\circ 33' 50.81'' \\ 22^\circ 13' 23.52'' \end{cases}$	$\begin{cases} 34^\circ 9' 8.91'' \\ 30^\circ 16' 2.76'' \\ 22^\circ 28' 41.68'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 44^\circ 45' 31.95'' \\ 31^\circ 27' 46.44'' \end{cases}$	$\begin{cases} 43^\circ 59' 41.32'' \\ 45^\circ 14' 34.29'' \\ 24^\circ 43' 50.08'' \end{cases}$	$\begin{cases} 32^\circ 26' 24.22'' \\ 34^\circ 43' 55.60'' \\ 31^\circ 27' 46.34'' \end{cases}$	9.94747	
116. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$\begin{cases} 22^\circ 45' 46.37'' \\ 24^\circ 28' 3.70'' \\ 21^\circ 45' 47.46'' \end{cases}$	$\begin{cases} 28^\circ 16' 10.80'' \\ 33^\circ 48' 59.21'' \\ 22^\circ 11' 35.74'' \end{cases}$	$\begin{cases} 34^\circ 25' 13.33'' \\ 31^\circ 27' 46.44'' \\ 22^\circ 28' 41.68'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 44^\circ 45' 31.95'' \\ 30^\circ 16' 2.76'' \end{cases}$	$\begin{cases} 43^\circ 13' 42.22'' \\ 45^\circ 39' 43.55'' \\ 24^\circ 33' 54.85'' \end{cases}$	$\begin{cases} 32^\circ 12' 15.21'' \\ 36^\circ 10' 42.75'' \\ 30^\circ 15' 20.49'' \end{cases}$	9.934566	
117. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$	$\begin{cases} 23^\circ 50' 32.39'' \\ 22^\circ 27' 24.92'' \\ 20^\circ 35' 48.29'' \end{cases}$	$\begin{cases} 31^\circ 37' 31.51'' \\ 34^\circ 22' 51.03'' \\ 24^\circ 27' 0.33'' \end{cases}$	$\begin{cases} 35^\circ 1' 20.66'' \\ 29^\circ 24' 59.22'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 43^\circ 5' 34.75'' \\ 41^\circ 35' 27.44'' \\ 28^\circ 57' 29.27'' \end{cases}$	$\begin{cases} 44^\circ 52' 55.77'' \\ 42^\circ 10' 10.96'' \\ 26^\circ 46' 57.36'' \end{cases}$	$\begin{cases} 36^\circ 13' 5.68'' \\ 32^\circ 46' 26.66'' \\ 29^\circ 3' 34.06'' \end{cases}$	10.13696	
118. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$	$\begin{cases} 23^\circ 50' 32.39'' \\ 22^\circ 11' 35.74'' \\ 20^\circ 48' 36.44'' \end{cases}$	$\begin{cases} 31^\circ 37' 31.51'' \\ 34^\circ 16' 39.91'' \\ 24^\circ 28' 3.70'' \end{cases}$	$\begin{cases} 34^\circ 54' 43.65'' \\ 28^\circ 57' 29.27'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 43^\circ 20' 56'' \\ 41^\circ 35' 27.44'' \\ 29^\circ 24' 59.22'' \end{cases}$	$\begin{cases} 45^\circ 7' 4.23'' \\ 42^\circ 1' 8.27'' \\ 26^\circ 50' 14.24'' \end{cases}$	$\begin{cases} 36^\circ 17' 50.69'' \\ 32^\circ 15' 16.27'' \\ 29^\circ 29' 45.45'' \end{cases}$	10.13696	
119. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.0188. \end{cases}$	$\begin{cases} 24^\circ 0' 35.31'' \\ 24^\circ 27' 0.33'' \\ 20^\circ 35' 48.29'' \end{cases}$	$\begin{cases} 30^\circ 24' 38.60'' \\ 34^\circ 22' 51.03'' \\ 22^\circ 27' 24.92'' \end{cases}$	$\begin{cases} 35^\circ 1' 20.66'' \\ 31^\circ 37' 17.14'' \\ 22^\circ 42' 40.72'' \end{cases}$	$\begin{cases} 43^\circ 5' 34.75'' \\ 43^\circ 16' 43.97'' \\ 27^\circ 58' 4.26'' \end{cases}$	$\begin{cases} 43^\circ 51' 37.22'' \\ 44^\circ 12' 36.86'' \\ 24^\circ 36' 32.72'' \end{cases}$	$\begin{cases} 34^\circ 43' 30.92'' \\ 35^\circ 49' 48.26'' \\ 28^\circ 1' 42.23'' \end{cases}$	9.956056	In (107), (119), (120) $V_1 = V_2 = V_3$ $A_1 = B_3$ $A_2 + A_3 = 180^\circ$ $B_1 + B_2 = 180^\circ$
120. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.0188. \end{cases}$	$\begin{cases} 24^\circ 0' 35.31'' \\ 22^\circ 14' 22.20'' \\ 22^\circ 17' 0.28'' \end{cases}$	$\begin{cases} 30^\circ 24' 38.60'' \\ 33^\circ 33' 50.81'' \\ 22^\circ 31' 9.76'' \end{cases}$	$\begin{cases} 34^\circ 0' 8.91'' \\ 27^\circ 58' 4.26'' \\ 22^\circ 42' 40.72'' \end{cases}$	$\begin{cases} 45^\circ 14' 28.04'' \\ 43^\circ 16' 43.97'' \\ 31^\circ 37' 17.14'' \end{cases}$	$\begin{cases} 46^\circ 8' 22.78'' \\ 42^\circ 57' 0.90'' \\ 24^\circ 59' 52.83'' \end{cases}$	$\begin{cases} 35^\circ 28' 14.10'' \\ 31^\circ 29' 26.40'' \\ 31^\circ 34' 42.73'' \end{cases}$	9.956056	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
121. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=4.0188 \\ c=3.9794, & z=4.2893 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 1' 31.91" 22° 31' 9.76" 22° 17' 0.28"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 35° 17' 35.48" 31° 54' 8.31" 31° 25' 45.34"	9.9345675	In (116), (121), (122) $V=V_1=V_2$ $B_1+B_2=180^\circ$
122. $\begin{cases} a=5.6794, & x=4.1941 \\ b=5.6318, & y=4.2893 \\ c=3.9794, & z=4.0188 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 28' 3.70" 20° 48' 36.44" 20° 1' 31.91"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 34° 37' 37.77" 28° 18' 54.19" 35° 43' 22.36"	9.934567	In (117), (118) the volumes are equal and the values of A supplementary.
123. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=4.0188 \\ c=4.1941, & z=3.9794 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 34' 0.17" 22° 29' 33.12" 20° 35' 48.29"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 37° 24' 0.02" 32° 13' 1.31" 28° 35' 34.23"	10.04542	
124. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=3.9794 \\ c=4.1941, & z=4.0188 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 34' 0.17" 22° 13' 23.52" 20° 48' 36.44"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 37° 29' 6.12" 31° 42' 29.17" 29° 0' 37.28"	10.042935	
125. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=4.1941 \\ c=6.0188, & z=3.9794 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 41' 24.64" 23° 46' 13.46" 20° 35' 48.29"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 36° 26' 19.21" 34° 7' 47.34" 28° 0' 33.33"	9.9481965	
126. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=3.9794 \\ c=4.0188, & z=4.1941 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 41' 24.64" 22° 14' 22.20" 21° 45' 47.46"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 36° 57' 24.79" 31° 14' 39.59" 30° 19' 59.15"	9.932784	
127. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=4.0188 \\ c=3.9794, & z=4.1941 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 42' 44.07" 22° 31' 9.76" 21° 45' 47.46"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 36° 46' 5.94" 31° 39' 4.54" 30° 12' 7.85"	9.913712	
128. $\begin{cases} a=5.6794, & x=4.2893 \\ b=5.6318, & y=4.1941 \\ c=3.9794, & z=4.0188 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 24° 42' 44.07" 23° 46' 53.71" 20° 48' 36.44"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 36° 20' 6.73" 34° 1' 41.19" 28° 17' 43.37"	9.9266415	
129. $\begin{cases} a=5.6794, & x=5.6318 \\ b=4.0188, & y=3.9794 \\ c=4.2893, & z=4.1941 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 42° 38' 39.62" 22° 11' 35.74" 23° 46' 13.46"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 70° 25' 58.44" 13° 8' 48.98" 13° 55' 35.62"	5.3144448	
130. $\begin{cases} a=5.6794, & x=5.6318 \\ b=4.0188, & y=4.1941 \\ c=4.2893, & z=3.9794 \end{cases}$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ 42° 38' 39.62" 23° 37' 55.51" 22° 14' 22.20"	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$ 70° 41' 21.10" 13° 41' 37.36" 13° 0' 50.23"	5.241143	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}a_5, \frac{1}{2}b_5, \frac{1}{2}c_5$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
131. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=3.9794, \end{cases}$	$\begin{cases} 44^\circ 45' 31.95'' \\ 24^\circ 28' 3.70'' \\ 23^\circ 46' 13.46'' \end{cases}$	$\begin{cases} 41^\circ 35' 27.44'' \\ 22^\circ 29' 33.12'' \\ 22^\circ 11' 35.74'' \end{cases}$	$\begin{cases} 43^\circ 44' 13.42'' \\ 24^\circ 34' 0.17'' \\ 22^\circ 28' 41.68'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 70^\circ 50' 12.18'' \\ 13^\circ 8' 27.08'' \\ 13^\circ 1' 29.00'' \end{cases}$	$\begin{cases} 70^\circ 34' 35.59'' \\ 14^\circ 0' 44.98'' \\ 13^\circ 41' 8.23'' \end{cases}$	5.247219	
132. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=3.9794, \end{cases}$	$\begin{cases} 44^\circ 45' 31.95'' \\ 23^\circ 46' 53.71'' \\ 24^\circ 27' 0.33'' \end{cases}$	$\begin{cases} 41^\circ 35' 27.44'' \\ 22^\circ 27' 24.92'' \\ 22^\circ 13' 23.52'' \end{cases}$	$\begin{cases} 43^\circ 54' 34.75'' \\ 23^\circ 50' 32.39'' \\ 22^\circ 28' 41.68'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 70^\circ 50' 12.18'' \\ 13^\circ 8' 27.08'' \\ 13^\circ 1' 29.00'' \end{cases}$	$\begin{cases} 70^\circ 44' 35.48'' \\ 13^\circ 40' 48.36'' \\ 13^\circ 59' 35.62'' \end{cases}$	5.242459	The tetrahedra from (129) to (134) are corrected for small angles by taking the average.
133. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=4.1941, \end{cases}$	$\begin{cases} 43^\circ 16' 43.97'' \\ 22^\circ 13' 23.52'' \\ 24^\circ 27' 0.33'' \end{cases}$	$\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 27' 24.92'' \\ 23^\circ 46' 53.71'' \end{cases}$	$\begin{cases} 43^\circ 54' 34.75'' \\ 22^\circ 24' 5.52'' \\ 24^\circ 0' 35.31'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 45' 46.37'' \\ 24^\circ 34' 0.17'' \end{cases}$	$\begin{cases} 70^\circ 35' 5.64'' \\ 13^\circ 16' 20.86'' \\ 13^\circ 55' 27.93'' \end{cases}$	$\begin{cases} 70^\circ 25' 44.83'' \\ 13^\circ 8' 31.16'' \\ 14^\circ 13' 47.61'' \end{cases}$	5.314359	
134. $\begin{cases} a=5.6794, \\ b=4.0188, \\ c=4.1941, \end{cases}$	$\begin{cases} 43^\circ 16' 43.97'' \\ 24^\circ 20' 44.08'' \\ 22^\circ 14' 22.20'' \end{cases}$	$\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 31' 9.76'' \\ 23^\circ 37' 55.51'' \end{cases}$	$\begin{cases} 45^\circ 14' 28.04'' \\ 22^\circ 24' 5.52'' \\ 24^\circ 0' 35.31'' \end{cases}$	$\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 42' 40.72'' \\ 22^\circ 24' 5.52'' \end{cases}$	$\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 42' 40.72'' \\ 22^\circ 24' 5.52'' \end{cases}$	$\begin{cases} 70^\circ 52' 51.89'' \\ 13^\circ 9' 21.25'' \\ 13^\circ 41' 49.68'' \end{cases}$	$\begin{cases} 70^\circ 40' 7.64'' \\ 14^\circ 1' 16.07'' \\ 13^\circ 0' 15.65'' \end{cases}$	5.245816	
135. $\begin{cases} a=3.4567, \\ b=3.004518, \\ c=2.8993, \end{cases}$	$\begin{cases} 50^\circ 40' 48.45'' \\ 40^\circ 5' 35.35'' \\ 38^\circ 11' 17.29'' \end{cases}$	$\begin{cases} 34^\circ 11' 43.79'' \\ 23^\circ 21' 49.55'' \\ 21^\circ 58' 47.78'' \end{cases}$	$\begin{cases} 28^\circ 26' 53.16'' \\ 28^\circ 20' 53.16'' \\ 19^\circ 14' 30.42'' \end{cases}$	$\begin{cases} 27^\circ 55' 36.87'' \\ 20^\circ 4' 41.13'' \\ 27^\circ 21' 23.05'' \end{cases}$	$\begin{cases} 27^\circ 55' 36.87'' \\ 20^\circ 4' 41.13'' \\ 27^\circ 21' 23.05'' \end{cases}$	$\begin{cases} 38^\circ 25' 21.76'' \\ 24^\circ 40' 38.14'' \\ 23^\circ 10' 1.34'' \end{cases}$	$\begin{cases} 52^\circ 10' 5.38'' \\ 38^\circ 25' 21.76'' \\ 36^\circ 54' 44.96'' \end{cases}$	4.656314	$X-A=Y-B=Z-C$ $a_3=b_3; A=Y$
136. $\begin{cases} a=3.46882, \\ b=3.243232, \\ c=3.952198, \end{cases}$	$\begin{cases} 22^\circ 56' 38.41'' \\ 24^\circ 28' 23.73'' \\ 30^\circ 0' 55.38'' \end{cases}$	$\begin{cases} 26^\circ 20' 40.57'' \\ 28^\circ 19' 37.53'' \\ 36^\circ 50' 5.46'' \end{cases}$	$\begin{cases} 31^\circ 30' 27.09'' \\ 29^\circ 31' 42.08'' \\ 40^\circ 4' 45.41'' \end{cases}$	$\begin{cases} 28^\circ 41' 30.81'' \\ 26^\circ 58' 36.18'' \\ 34^\circ 7' 37.35'' \end{cases}$	$\begin{cases} 28^\circ 41' 30.81'' \\ 26^\circ 58' 36.18'' \\ 34^\circ 7' 37.35'' \end{cases}$	$\begin{cases} 32^\circ 20' 52.34'' \\ 30^\circ 6' 10.24'' \\ 47^\circ 14' 42.10'' \end{cases}$	$\begin{cases} 27^\circ 51' 28.14'' \\ 30^\circ 6' 10.24'' \\ 42^\circ 45' 17.90'' \end{cases}$	4	$A+V=B+Y; A-V=C-Z; B=Y; C+Z=180^\circ; V$ a max. where $a+x=b+y=6.348864$ $a-x=c-z=0.5889$
137. $\begin{cases} a=4.9916, \\ b=4.8548, \\ c=3.5152, \end{cases}$	$\begin{cases} 34^\circ 5' 19.96'' \\ 34^\circ 59' 23.99'' \\ 20^\circ 28' 51.49'' \end{cases}$	$\begin{cases} 33^\circ 10' 13.54'' \\ 33^\circ 41' 39.78'' \\ 20^\circ 32' 31.41'' \end{cases}$	$\begin{cases} 35^\circ 49' 28.73'' \\ 36^\circ 22' 1.66'' \\ 21^\circ 18' 10.40'' \end{cases}$	$\begin{cases} 34^\circ 28' 4.69'' \\ 34^\circ 36' 29.64'' \\ 20^\circ 27' 44.80'' \end{cases}$	$\begin{cases} 34^\circ 28' 4.69'' \\ 34^\circ 36' 29.64'' \\ 20^\circ 27' 44.80'' \end{cases}$	$\begin{cases} 40^\circ 42' 46.83'' \\ 41^\circ 5' 6.28'' \\ 22^\circ 25' 29.34'' \end{cases}$	$\begin{cases} 39^\circ 42' 16.83'' \\ 42^\circ 5' 36.28'' \\ 21^\circ 58' 44.36'' \end{cases}$	8.595858	
138. $\begin{cases} a=4.9916, \\ b=4.8548, \\ c=3.5152, \end{cases}$	$\begin{cases} 34^\circ 5' 19.96'' \\ 21^\circ 49' 53.13'' \\ 30^\circ 40' 56.98'' \end{cases}$	$\begin{cases} 33^\circ 10' 13.54'' \\ 28^\circ 59' 17.05'' \\ 22^\circ 22' 20.79'' \end{cases}$	$\begin{cases} 30^\circ 19' 45.97'' \\ 20^\circ 27' 44.80'' \\ 21^\circ 18' 10.40'' \end{cases}$	$\begin{cases} 45^\circ 47' 46.08'' \\ 34^\circ 36' 29.64'' \\ 30^\circ 22' 1.66'' \end{cases}$	$\begin{cases} 45^\circ 47' 46.08'' \\ 34^\circ 36' 29.64'' \\ 30^\circ 22' 1.66'' \end{cases}$	$\begin{cases} 49^\circ 17' 13.17'' \\ 33^\circ 49' 18.60'' \\ 25^\circ 4' 56.90'' \end{cases}$	$\begin{cases} 43^\circ 48' 47.64'' \\ 23^\circ 59' 48.63'' \\ 35^\circ 25' 19.94'' \end{cases}$	8.595857	
139. $\begin{cases} a=4.9916, \\ b=4.8548, \\ c=3.5152, \end{cases}$	$\begin{cases} 22^\circ 36' 43.56'' \\ 34^\circ 59' 23.99'' \\ 29^\circ 18' 11.27'' \end{cases}$	$\begin{cases} 20^\circ 27' 44.80'' \\ 29^\circ 37' 21.89'' \\ 20^\circ 32' 31.41'' \end{cases}$	$\begin{cases} 31^\circ 3' 26.84'' \\ 36^\circ 22' 1.66'' \\ 22^\circ 11' 2.51'' \end{cases}$	$\begin{cases} 34^\circ 28' 4.69'' \\ 44^\circ 12' 13.93'' \\ 33^\circ 10' 13.54'' \end{cases}$	$\begin{cases} 34^\circ 28' 4.69'' \\ 44^\circ 12' 13.93'' \\ 33^\circ 10' 13.54'' \end{cases}$	$\begin{cases} 33^\circ 49' 18.60'' \\ 48^\circ 54' 53.72'' \\ 24^\circ 36' 51.15'' \end{cases}$	$\begin{cases} 26^\circ 30' 30.29'' \\ 46^\circ 9' 39.60'' \\ 32^\circ 35' 55.73'' \end{cases}$	8.595857	See next page.
140. $\begin{cases} a=4.9916, \\ b=4.8548, \\ c=3.4476, \end{cases}$	$\begin{cases} 34^\circ 17' 51.19'' \\ 35^\circ 12' 7.16'' \\ 20^\circ 54' 6.54'' \end{cases}$	$\begin{cases} 32^\circ 59' 41.69'' \\ 36^\circ 7' 16.98'' \\ 20^\circ 52' 25.74'' \end{cases}$	$\begin{cases} 35^\circ 35' 27.01'' \\ 36^\circ 7' 16.98'' \\ 20^\circ 52' 25.74'' \end{cases}$	$\begin{cases} 34^\circ 40' 5.98'' \\ 34^\circ 49' 43.07'' \\ 20^\circ 53' 1.33'' \end{cases}$	$\begin{cases} 34^\circ 40' 5.98'' \\ 34^\circ 49' 43.07'' \\ 20^\circ 53' 1.33'' \end{cases}$	$\begin{cases} 40^\circ 41' 27.16'' \\ 41^\circ 6' 24.24'' \\ 22^\circ 2' 2.18'' \end{cases}$	$\begin{cases} 39^\circ 43' 34.92'' \\ 42^\circ 4' 16.48'' \\ 22^\circ 22' 8.53'' \end{cases}$	8.596395	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}A', \frac{1}{2}B', \frac{1}{2}C'$	VOLUME.	REMARKS.
141. $\begin{cases} a=4.9916, & x=4.8206 \\ b=4.8548, & y=3.5152 \\ c=3.4476, & z=5.0258 \end{cases}$	$34^\circ 17' 51.19''$ $22^\circ 22' 20.79''$ $30^\circ 40' 50.98''$	$32^\circ 59' 41.69''$ $28^\circ 59' 17.05''$ $21^\circ 49' 53.13''$	$30^\circ 19' 45.97''$ $20^\circ 53' 1.33''$ $20^\circ 52' 25.74''$	$45^\circ 47' 46.08''$ $34^\circ 49' 43.07''$ $36^\circ 7' 16.98''$	$49^\circ 18' 32.84''$ $34^\circ 0' 59.80''$ $24^\circ 31' 19.15''$	$43^\circ 51' 35.11''$ $24^\circ 52' 5.12''$ $35^\circ 11' 31.67''$	8.596395	A set of ten tetrahedra having the same six lengths for their edges. $V_1 = V_2 = V_3$ $A_1 + A_2 = B_1 + B_3 = 180^\circ$
142. $\begin{cases} a=4.9916, & x=3.5152 \\ b=4.8548, & y=5.0258 \\ c=3.4476, & z=4.8206 \end{cases}$	$23^\circ 11' 2.51''$ $35^\circ 12' 7.16''$ $29^\circ 18' 11.27''$	$20^\circ 53' 1.33''$ $20^\circ 38' 21.89''$ $20^\circ 7' 46.86''$	$31^\circ 3' 26.84''$ $36^\circ 7' 16.98''$ $22^\circ 36' 43.56''$	$34^\circ 40' 5.98''$ $44^\circ 12' 13.93''$ $32^\circ 59' 41.69''$	$34^\circ 0' 59.80''$ $48^\circ 53' 35.76''$ $24^\circ 4' 17.76''$	$25^\circ 4' 8.33''$ $46^\circ 6' 56.94''$ $32^\circ 26' 19.08''$	8.596395	$B_2 = A_3$ $V_2 = V_3 = V_4$ $A_4 + A_6 = B_4 + B_6 = 180^\circ$
143. $\begin{cases} a=4.9916, & x=4.8206 \\ b=4.8548, & y=3.5152 \\ c=5.0258, & z=3.4476 \end{cases}$	$29^\circ 11' 10.80''$ $20^\circ 32' 31.41''$ $20^\circ 28' 51.49''$	$43^\circ 48' 45.10''$ $33^\circ 41' 39.78''$ $34^\circ 59' 23.90''$	$35^\circ 49' 28.73''$ $33^\circ 23' 1.09''$ $31^\circ 17' 39.03''$	$34^\circ 28' 4.69''$ $29^\circ 31' 10.17''$ $22^\circ 48' 13.81''$	$45^\circ 1' 21.50''$ $33^\circ 13' 3.73''$ $34^\circ 38' 13.13''$	$47^\circ 59' 3.02''$ $25^\circ 4' 11.62''$ $29^\circ 59' 10.08''$	8.697423	$B_2 = A_3$ $V_2 = V_3 = V_4$ $A_4 + A_6 = B_4 + B_6 = 180^\circ$
144. $\begin{cases} a=4.9916, & x=4.8206 \\ b=4.8548, & y=3.4476 \\ c=5.0258, & z=3.5152 \end{cases}$	$29^\circ 11' 10.80''$ $20^\circ 7' 46.86''$ $20^\circ 54' 6.54''$	$43^\circ 48' 45.10''$ $33^\circ 30' 26.45''$ $35^\circ 12' 7.16''$	$35^\circ 35' 27.01''$ $22^\circ 48' 13.81''$ $31^\circ 17' 39.03''$	$34^\circ 40' 5.98''$ $29^\circ 31' 10.17''$ $23^\circ 23' 1.09''$	$44^\circ 58' 38.50''$ $33^\circ 12' 28.71''$ $34^\circ 50' 49.43''$	$47^\circ 57' 45.17''$ $24^\circ 30' 31.56''$ $25^\circ 34' 9.99''$	8.697421	$B_2 = A_3$ $V_2 = V_3 = V_4$ $A_7 + A_8 = B_7 + B_{10} = 180^\circ$
145. $\begin{cases} a=4.9916, & x=3.5152 \\ b=4.8548, & y=5.0258 \\ c=4.8206, & z=3.4476 \end{cases}$	$21^\circ 18' 10.40''$ $30^\circ 47' 44.97''$ $20^\circ 28' 51.49''$	$22^\circ 9' 57.78''$ $33^\circ 41' 39.78''$ $28^\circ 45' 52.84''$	$35^\circ 49' 28.73''$ $46^\circ 12' 4.72''$ $34^\circ 5' 19.96''$	$30^\circ 26' 22.19''$ $34^\circ 36' 29.64''$ $21^\circ 37' 57.50''$	$34^\circ 33' 55.67''$ $44^\circ 58' 40.52''$ $33^\circ 1' 33.68''$	$24^\circ 36' 1.26''$ $50^\circ 11' 47.15''$ $23^\circ 34' 21.47''$	8.498537	Taking a, b, c, x, y, z to have the meaning of (135)
146. $\begin{cases} a=4.9916, & x=3.4476 \\ b=4.8548, & y=5.0258 \\ c=4.8206, & z=3.5152 \end{cases}$	$20^\circ 52' 25.74''$ $30^\circ 47' 44.97''$ $20^\circ 54' 6.54''$	$21^\circ 37' 57.50''$ $33^\circ 30' 26.45''$ $28^\circ 45' 52.84''$	$35^\circ 35' 27.01''$ $46^\circ 12' 4.72''$ $34^\circ 5' 19.96''$	$30^\circ 26' 22.19''$ $34^\circ 36' 29.64''$ $22^\circ 9' 57.78''$	$34^\circ 21' 3.65''$ $45^\circ 1' 19.48''$ $33^\circ 12' 16.30''$	$24^\circ 3' 26.87''$ $50^\circ 13' 6.75''$ $24^\circ 5' 32.85''$	8.498537	$a^2 + x^2 = 2(\rho^2 + s^2)$ $3a^2 + 2x^2 = 3\rho^2 + 2s^2$
147. $\begin{cases} a=7.8293, & x=4.9316 \\ b=7.1131, & y=5.5722 \\ c=8.7543, & z=5.8566 \end{cases}$	$17^\circ 8' 33.24''$ $19^\circ 22' 54.45''$ $22^\circ 49' 45.12''$	$25^\circ 31' 33.85''$ $30^\circ 41' 32.71''$ $39^\circ 48' 49.52''$	$36^\circ 28' 42.17''$ $30^\circ 44' 43.67''$ $45^\circ 15' 30.45''$	$30^\circ 48' 16.03''$ $27^\circ 35' 56.31''$ $33^\circ 43' 42.48''$	$35^\circ 6' 31.40''$ $30^\circ 31' 19.47''$ $50^\circ 6' 17.04''$	$25^\circ 32' 39.81''$ $29^\circ 56' 0.72''$ $40^\circ 32' 25.45''$	28.61265	In the four tetrahedra, (147) to (150), $a=7.8293, x=4.9316, b+y=a+x, c=z=a-x$; and (150) is the one of maximum volume under these conditions.
148. $\begin{cases} a=7.8293, & x=4.9316 \\ b=7.1131, & y=5.6488 \\ c=8.7303, & z=5.8326 \end{cases}$	$17^\circ 11' 37.39''$ $19^\circ 22' 54.45''$ $22^\circ 49' 10.52''$	$25^\circ 25' 28.08''$ $30^\circ 20' 26.98''$ $39^\circ 27' 57.69''$	$36^\circ 50' 22.50''$ $31^\circ 19' 4.94''$ $45^\circ 31' 52.97''$	$30^\circ 49' 41.45''$ $27^\circ 16' 29.64''$ $33^\circ 15' 26.98''$	$35^\circ 29' 4.48''$ $30^\circ 31' 19.47''$ $49^\circ 59' 24.44''$	$25^\circ 32' 56.82''$ $30^\circ 30' 41.83''$ $40^\circ 3' 16.78''$	28.62442	
149. $\begin{cases} a=7.8293, & x=4.9316 \\ b=7.1121, & y=5.6488 \\ c=8.7285, & z=5.8308 \end{cases}$	$17^\circ 11' 52.27''$ $19^\circ 22' 54.45''$ $22^\circ 48' 47.56''$	$25^\circ 25' 36.60''$ $30^\circ 20' 17.87''$ $39^\circ 27' 10.51''$	$36^\circ 50' 54.57''$ $31^\circ 19' 53.64''$ $45^\circ 31' 27.95''$	$30^\circ 50' 3.73''$ $27^\circ 16' 39.78''$ $33^\circ 14' 29.76''$	$35^\circ 30' 4.64''$ $30^\circ 31' 48.27''$ $49^\circ 58' 17.30''$	$25^\circ 33' 31.51''$ $30^\circ 31' 47.88''$ $40^\circ 1' 44.17''$	28.62443	
150. $\begin{cases} a=7.8293, & x=4.9316 \\ b=7.112089, & y=5.648811 \\ c=8.728483, & z=5.830783 \end{cases}$	$17^\circ 11' 52.41''$ $19^\circ 22' 54.01''$ $22^\circ 48' 47.33''$	$25^\circ 25' 36.67''$ $30^\circ 20' 17.76''$ $39^\circ 27' 10.05''$	$36^\circ 50' 54.91''$ $31^\circ 19' 54.15''$ $45^\circ 31' 27.74''$	$30^\circ 50' 3.94''$ $27^\circ 16' 39.85''$ $33^\circ 14' 29.18''$	$35^\circ 30' 5.20''$ $30^\circ 31' 48.52''$ $49^\circ 58' 16.68''$	$25^\circ 33' 31.84''$ $30^\circ 31' 48.52''$ $40^\circ 1' 43.32''$	28.624435	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
151. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases} \begin{matrix} x=4.8370 \\ y=7.5278 \\ z=6.1874 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 16°49'25.87" 25°8'46.85" 19°57'47.85"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 19°54'24.82" 31°18'25.89" 27°48'15.68"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 38°43'46.26" 42°35'53.04" 33°49'51.15"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 37°2'57.47" 39°20'42.98" 27°29'42.14"	39°12' 3.07" 43°35' 2.42" 34° 4'41.47"	23° 2'11.94" 44°57'41.02" 28°16' 8.41"	38.5523	
152. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{matrix} \begin{matrix} x=4.8370 \\ y=6.1874 \\ z=7.5278 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 16°49'25.87" 20°16'30.92" 24°37'29.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 19°54'24.82" 29°44'25.39" 29°1'10.18"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 35°38' 4.95" 27°29'42.14" 33°49'51.15"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 40°42'18.90" 39°20'42.98" 42°35'53.04"	41° 5'26.07" 39°37'49.63" 37°41'16.24"	27°27'13.26" 29°28'49.66" 43°23'24.62"	38.99001	Set of thirty tetrahedra having all the same six lengths for their edges. (151) to (156) is Group I., in which 9.4106, 4.8370 are opposite.
153. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{matrix} \begin{matrix} x=4.8370 \\ y=8.0746 \\ z=6.1874 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 17°7' 4.08" 21°48'15.68" 19°57'47.85"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 18°21'48.60" 31°18'25.89" 25°8'46.85"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 38°43'46.26" 46°41' 6.12" 30°33'22.03"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 37°2'57.47" 42°10'33.89" 24°57' 5.28"	38°50'33.15" 48°14'57.84" 29°42'21.74"	21° 0' 2.88" 50°31'14.11" 24°52'50.62"	35.81829	(157) to (162) is Group II., in which 9.4106, 6.1874 are opposite. (163) to (168) is Group III., in which 9.4106, 7.5278 are opposite. (169) to (174) is Group IV., in which 9.4106, 8.0746 are opposite; and lastly, (175) to (180) is Group V., in which the two longest edges 9.4106, 8.5602 are opposite. The average volume is considerably the least in this group; indeed, 5 out of the 6 least are in this group.
154. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{matrix} \begin{matrix} x=4.8370 \\ y=6.1874 \\ z=8.0746 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 17°7' 4.08" 20°29'44.11" 26°34'18.74"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 18°21'48.60" 29°44'25.39" 26°28'18.40"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 34°24'59.46" 24°57' 5.28" 30°33'22.03"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 43°1'57.49" 42°10'33.89" 46°41' 6.12"	42°51'44.14" 42°11'54.78" 34°43'10.94"	22°16'40.71" 27°26'35.89" 40°52'52.44"	37.21727	
155. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{matrix} \begin{matrix} x=4.8370 \\ y=7.5278 \\ z=8.0746 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 16°49'58.84" 26°28'18.40" 26°34'18.74"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 17°57' 6.85" 29°44'25.39" 20°29'44.11"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 34°24'59.46" 32°56'14.05" 22°34'53.39"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 43°1'57.49" 50°35' 7.77" 39° 6'39.10"	41°43'59.80" 51° 8'26.42" 24°32'17.15"	21°14'48.00" 38°15'37.49" 38°34'57.26"	35.83271	
156. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{matrix} \begin{matrix} x=4.8370 \\ y=8.0746 \\ z=7.5278 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 16°49'58.84" 29°1'10.18" 24°37'29.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 17°57' 6.85" 29°44'25.39" 20°16'30.92"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 35°38' 4.95" 39° 6'39.10" 22°34'53.39"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 40°42'18.90" 50°35' 7.77" 32°56'14.05"	37°36'20.76" 53°11'59.38" 23°11'29.52"	20°23' 8.27" 45°57' 4.25" 31°35'24.39"	34.86761	
157. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{matrix} \begin{matrix} x=6.1874 \\ y=7.5278 \\ z=4.8370 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 21°46'39.73" 25°8'46.85" 15°22'57.43"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 27°29'42.14" 32°25'44.16" 27°48'15.68"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 42°11'18.41" 42°35'53.04" 32°1'21.46"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 37°2'57.47" 36°12' 7.81" 19°54'24.82"	43° 2'14.82" 40°42'40.59" 32°10' 9.35"	31°43'55.28" 43°41'28.89" 20°48'57.02"	38.16771	
158. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{matrix} \begin{matrix} x=6.1874 \\ y=4.8370 \\ z=7.5278 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 21°46'39.73" 15°27'54.53" 24°37'29.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 27°29'42.14" 29°44'25.39" 29°32'53.45"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 35°38' 4.95" 19°54'24.82" 32°1'21.46"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 44°59'12.02" 36°12' 7.81" 42°35'53.04"	45°44'58.47" 36°10'16.71" 35°49'12.37"	32°28'40.22" 21°15'38.66" 42°29' 2.81"	38.24435	
159. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{matrix} \begin{matrix} x=6.1874 \\ y=8.0746 \\ z=7.5278 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 22°34'53.39" 29°32'53.45" 24°37'29.66"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 23°16'55.55" 29°44'25.39" 15°27'54.53"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 35°38' 4.95" 35°41'13.44" 16°49'58.84"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 44°59'12.02" 50°35' 7.77" 31°1'51.03"	41°58'43.52" 51°20'27.93" 17°47'57.02"	27°39'39.12" 42° 9' 3.20" 30°44' 1.05"	34.71445	
160. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{matrix} \begin{matrix} x=6.1874 \\ y=7.5278 \\ z=8.0746 \end{matrix}$	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ 22°34'53.39" 26°18' 8.81" 26°34'18.74"	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ 23°16'55.55" 29° 0'41.86" 15°20'53.15"	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ 34°24'59.46" 31°1'51.03" 16°49'58.84"	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ 48°20'58.04" 50°35' 7.77" 35°41'13.44"	46°41'19.92" 49°47' 7.25" 18°12' 0.79"	28°47'20.53" 35°31' 2.79" 36° 7'58.78"	34.84775	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_0, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_0, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_0, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_0, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
161. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{cases}$ $\begin{cases} x=6.1874 \\ y=4.8370 \\ z=8.0746 \end{cases}$	$\begin{cases} 22^{\circ}19'58.79'' \\ 15^{\circ}20'53.15'' \\ 26^{\circ}34'18.74'' \end{cases}$	$\begin{cases} 24^{\circ}57'5.28'' \\ 29^{\circ}0'41.80'' \\ 26^{\circ}18'8.81'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 18^{\circ}21'48.60'' \\ 29^{\circ}23'39.31'' \end{cases}$	$\begin{cases} 48^{\circ}20'58.04'' \\ 38^{\circ}16'21.90'' \\ 40^{\circ}41'6.12'' \end{cases}$	$\begin{cases} 48^{\circ}2'39.21'' \\ 38^{\circ}25'2.38'' \\ 32^{\circ}53'20.84'' \end{cases}$	$\begin{cases} 30^{\circ}42'22.53'' \\ 19^{\circ}48'30.39'' \\ 45^{\circ}56'19.15'' \end{cases}$	36.24991	
162. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=7.5278, \end{cases}$ $\begin{cases} x=6.1874 \\ y=8.0746 \\ z=4.8370 \end{cases}$	$\begin{cases} 22^{\circ}19'58.79'' \\ 27^{\circ}48'15.68'' \\ 15^{\circ}22'57.43'' \end{cases}$	$\begin{cases} 24^{\circ}57'5.28'' \\ 32^{\circ}25'44.16'' \\ 25^{\circ}8'46.85'' \end{cases}$	$\begin{cases} 42^{\circ}11'18.41'' \\ 46^{\circ}41'6.12'' \\ 29^{\circ}23'39.31'' \end{cases}$	$\begin{cases} 37^{\circ}2'57.47'' \\ 38^{\circ}16'21.90'' \\ 18^{\circ}28'48.60'' \end{cases}$	$\begin{cases} 40^{\circ}40'25.21'' \\ 44^{\circ}20'54.69'' \\ 29^{\circ}5'5.59'' \end{cases}$	$\begin{cases} 28^{\circ}49'30.03'' \\ 48^{\circ}41'52.23'' \\ 18^{\circ}57'56.75'' \end{cases}$	36.04062	
163. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$ $\begin{cases} x=7.5278 \\ y=4.8370 \\ z=6.1874 \end{cases}$	$\begin{cases} 26^{\circ}51'29.37'' \\ 15^{\circ}27'54.53'' \\ 19^{\circ}57'47.85'' \end{cases}$	$\begin{cases} 42^{\circ}35'53.04'' \\ 31^{\circ}18'25.89'' \\ 29^{\circ}32'53.45'' \end{cases}$	$\begin{cases} 38^{\circ}43'46.26'' \\ 19^{\circ}54'24.88'' \\ 29^{\circ}55'16.88'' \end{cases}$	$\begin{cases} 44^{\circ}59'12.02'' \\ 33^{\circ}13'13.75'' \\ 27^{\circ}29'42.14'' \end{cases}$	$\begin{cases} 53^{\circ}52'24.11'' \\ 30^{\circ}24'22.53'' \\ 28^{\circ}45'46.17'' \end{cases}$	$\begin{cases} 50^{\circ}46'23.46'' \\ 19^{\circ}19'52.33'' \\ 25^{\circ}38'1.68'' \end{cases}$	35.03920	
164. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=8.0746, \end{cases}$ $\begin{cases} x=7.5278 \\ y=6.1874 \\ z=4.8370 \end{cases}$	$\begin{cases} 26^{\circ}51'29.37'' \\ 20^{\circ}16'30.92'' \\ 15^{\circ}22'57.43'' \end{cases}$	$\begin{cases} 42^{\circ}35'53.04'' \\ 32^{\circ}25'44.16'' \\ 29^{\circ}1'10.18'' \end{cases}$	$\begin{cases} 42^{\circ}11'18.41'' \\ 27^{\circ}29'42.14'' \\ 29^{\circ}55'16.88'' \end{cases}$	$\begin{cases} 40^{\circ}42'18.90'' \\ 33^{\circ}13'13.75'' \\ 19^{\circ}54'24.82'' \end{cases}$	$\begin{cases} 52^{\circ}47'23.77'' \\ 31^{\circ}37'28.69'' \\ 28^{\circ}24'24.98'' \end{cases}$	$\begin{cases} 50^{\circ}17'42.86'' \\ 26^{\circ}13'20.13'' \\ 19^{\circ}17'43.82'' \end{cases}$	35.43771	
165. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$ $\begin{cases} x=7.5278 \\ y=8.0746 \\ z=4.8370 \end{cases}$	$\begin{cases} 29^{\circ}23'39.31'' \\ 29^{\circ}1'10.18'' \\ 15^{\circ}22'57.43'' \end{cases}$	$\begin{cases} 32^{\circ}56'14.05'' \\ 32^{\circ}25'44.16'' \\ 20^{\circ}16'30.92'' \end{cases}$	$\begin{cases} 42^{\circ}11'18.41'' \\ 39^{\circ}6'39.10'' \\ 22^{\circ}19'58.79'' \end{cases}$	$\begin{cases} 40^{\circ}42'18.90'' \\ 38^{\circ}16'21.90'' \\ 17^{\circ}57'6.85'' \end{cases}$	$\begin{cases} 46^{\circ}47'58.07'' \\ 39^{\circ}26'48.45'' \\ 22^{\circ}24'10.86'' \end{cases}$	$\begin{cases} 40^{\circ}48'11.14'' \\ 39^{\circ}27'37.47'' \\ 18^{\circ}8'17.65'' \end{cases}$	35.66563	
166. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$ $\begin{cases} x=7.5278 \\ y=4.8370 \\ z=8.0746 \end{cases}$	$\begin{cases} 29^{\circ}23'39.31'' \\ 13^{\circ}40'10.53'' \\ 26^{\circ}34'18.74'' \end{cases}$	$\begin{cases} 32^{\circ}56'14.05'' \\ 29^{\circ}0'41.80'' \\ 17^{\circ}59'20.22'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 17^{\circ}57'6.85'' \\ 22^{\circ}19'58.79'' \end{cases}$	$\begin{cases} 58^{\circ}20'29.25'' \\ 38^{\circ}16'21.90'' \\ 39^{\circ}6'39.10'' \end{cases}$	$\begin{cases} 60^{\circ}43'4.13'' \\ 34^{\circ}7'3.07'' \\ 20^{\circ}0'54.13'' \end{cases}$	$\begin{cases} 46^{\circ}8'23.03'' \\ 16^{\circ}13'28.91'' \\ 34^{\circ}35'56.70'' \end{cases}$	30.50267	See previous page.
167. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=4.8370, \end{cases}$ $\begin{cases} x=7.5278 \\ y=6.1874 \\ z=8.0746 \end{cases}$	$\begin{cases} 30^{\circ}33'22.03'' \\ 17^{\circ}59'20.22'' \\ 29^{\circ}32'53.45'' \end{cases}$	$\begin{cases} 31^{\circ}1'51.03'' \\ 29^{\circ}0'41.80'' \\ 13^{\circ}40'10.53'' \end{cases}$	$\begin{cases} 34^{\circ}24'59.46'' \\ 23^{\circ}16'55.53'' \\ 17^{\circ}7'4.08'' \end{cases}$	$\begin{cases} 58^{\circ}20'29.25'' \\ 42^{\circ}19'33.89'' \\ 35^{\circ}41'13.44'' \end{cases}$	$\begin{cases} 60^{\circ}14'54.68'' \\ 36^{\circ}53'9.33'' \\ 15^{\circ}39'38.41'' \end{cases}$	$\begin{cases} 45^{\circ}8'23.87'' \\ 21^{\circ}4'13.41'' \\ 33^{\circ}4'53.18'' \end{cases}$	30.52649	
168. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=4.8370, \end{cases}$ $\begin{cases} x=7.5278 \\ y=8.0746 \\ z=6.1874 \end{cases}$	$\begin{cases} 30^{\circ}33'22.03'' \\ 29^{\circ}32'53.45'' \\ 19^{\circ}57'47.85'' \end{cases}$	$\begin{cases} 31^{\circ}1'51.03'' \\ 31^{\circ}18'25.89'' \\ 15^{\circ}27'54.53'' \end{cases}$	$\begin{cases} 38^{\circ}43'46.26'' \\ 35^{\circ}41'13.44'' \\ 17^{\circ}7'4.08'' \end{cases}$	$\begin{cases} 44^{\circ}59'12.02'' \\ 42^{\circ}19'33.89'' \\ 23^{\circ}16'55.53'' \end{cases}$	$\begin{cases} 47^{\circ}31'26.30'' \\ 41^{\circ}19'31.97'' \\ 17^{\circ}31'7.57'' \end{cases}$	$\begin{cases} 40^{\circ}20'14.03'' \\ 37^{\circ}37'19.89'' \\ 23^{\circ}9'53.99'' \end{cases}$	35.29027	
169. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$ $\begin{cases} x=8.0746 \\ y=7.5278 \\ z=4.8370 \end{cases}$	$\begin{cases} 32^{\circ}1'21.46'' \\ 26^{\circ}28'18.40'' \\ 15^{\circ}22'57.43'' \end{cases}$	$\begin{cases} 39^{\circ}6'39.10'' \\ 32^{\circ}25'44.16'' \\ 20^{\circ}29'44.11'' \end{cases}$	$\begin{cases} 42^{\circ}11'18.41'' \\ 33^{\circ}56'14.05'' \\ 21^{\circ}46'30.73'' \end{cases}$	$\begin{cases} 43^{\circ}1'57.49'' \\ 36^{\circ}12'7.81'' \\ 17^{\circ}57'6.85'' \end{cases}$	$\begin{cases} 54^{\circ}11'20.64'' \\ 32^{\circ}31'44.02'' \\ 20^{\circ}32'10.44'' \end{cases}$	$\begin{cases} 50^{\circ}39'38.21'' \\ 30^{\circ}14'46.57'' \\ 16^{\circ}57'10.69'' \end{cases}$	33.25172	
170. $\begin{cases} a=9.4106, \\ b=8.5602, \\ c=6.1874, \end{cases}$ $\begin{cases} x=8.0746 \\ y=4.8370 \\ z=7.5278 \end{cases}$	$\begin{cases} 32^{\circ}1'21.46'' \\ 13^{\circ}40'10.53'' \\ 24^{\circ}37'29.66'' \end{cases}$	$\begin{cases} 39^{\circ}6'39.10'' \\ 29^{\circ}44'25.39'' \\ 17^{\circ}59'20.22'' \end{cases}$	$\begin{cases} 35^{\circ}38'4.95'' \\ 36^{\circ}12'7.81'' \\ 32^{\circ}56'14.05'' \end{cases}$	$\begin{cases} 58^{\circ}20'29.25'' \\ 36^{\circ}12'7.81'' \\ 32^{\circ}56'14.05'' \end{cases}$	$\begin{cases} 65^{\circ}34'54.56'' \\ 26^{\circ}42'55.93'' \\ 16^{\circ}36'17.22'' \end{cases}$	$\begin{cases} 57^{\circ}3'40.65'' \\ 15^{\circ}53'34.68'' \\ 25^{\circ}7'49.90'' \end{cases}$	26.37837	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
171. $\begin{cases} a=9.4106, & x=8.0746 \\ b=8.5602, & y=6.1874 \\ c=4.8370, & z=7.5278 \end{cases}$	$33^\circ 49' 51.15''$ $33^\circ 41' 13.44''$ $23^\circ 50' 22.22''$ $24^\circ 37' 29.66''$	$35^\circ 41' 13.44''$ $29^\circ 44' 25.39''$ $13^\circ 40' 10.53''$	$33^\circ 38' 4.95''$ $23^\circ 16' 55.53''$ $16^\circ 49' 25.87''$	$58^\circ 20' 29.25''$ $39^\circ 20' 42.98''$ $31^\circ 1' 51.03''$	$64^\circ 19' 11.26''$ $29^\circ 30' 8.69''$ $13^\circ 35' 47.76''$	$54^\circ 43' 24.15''$ $18^\circ 23' 44.40''$ $25^\circ 16' 49.78''$	27.25348	
172. $\begin{cases} a=9.4106, & x=8.0746 \\ b=8.5602, & y=7.5278 \\ c=4.8370, & z=6.1874 \end{cases}$	$33^\circ 49' 51.15''$ $26^\circ 18' 8.81''$ $19^\circ 57' 47.85''$	$35^\circ 41' 13.44''$ $31^\circ 18' 25.89''$ $15^\circ 20' 53.15''$	$38^\circ 43' 46.26''$ $31^\circ 1' 51.03''$ $16^\circ 49' 25.87''$	$48^\circ 20' 58.04''$ $39^\circ 20' 42.98''$ $33^\circ 44' 9.76''$	$55^\circ 20' 28.14''$ $33^\circ 44' 9.76''$ $16^\circ 21' 7.06''$	$49^\circ 50' 6.53''$ $28^\circ 55' 41.77''$ $21^\circ 34' 45.08''$	32.64297	
173. $\begin{cases} a=9.4106, & x=8.0746 \\ b=8.5602, & y=4.8370 \\ c=7.5278, & z=6.1874 \end{cases}$	$29^\circ 55' 16.88''$ $15^\circ 20' 53.15''$ $19^\circ 57' 47.85''$	$46^\circ 41' 6.12''$ $31^\circ 18' 25.89''$ $26^\circ 18' 8.81''$	$38^\circ 43' 46.26''$ $18^\circ 21' 48.66''$ $26^\circ 51' 29.37''$	$48^\circ 20' 58.04''$ $33^\circ 44' 9.76''$ $24^\circ 57' 5.28''$	$61^\circ 7' 16.25''$ $25^\circ 39' 32.96''$ $22^\circ 9' 6.51''$	$59^\circ 19' 14.66''$ $15^\circ 36' 15.18''$ $20^\circ 4' 7.82''$	29.0616	
174. $\begin{cases} a=9.4106, & x=8.0746 \\ b=8.5602, & y=6.1874 \\ c=7.5278, & z=4.8370 \end{cases}$	$29^\circ 55' 16.88''$ $20^\circ 29' 44.11''$ $15^\circ 22' 57.43''$	$46^\circ 41' 6.12''$ $32^\circ 25' 44.16''$ $26^\circ 28' 18.40''$	$42^\circ 11' 18.41''$ $26^\circ 57' 5.28''$ $26^\circ 51' 29.37''$	$43^\circ 1' 57.49''$ $33^\circ 13' 13.75''$ $18^\circ 21' 48.66''$	$58^\circ 40' 7.51''$ $27^\circ 21' 10.86''$ $23^\circ 0' 28.50''$	$57^\circ 55' 0.85''$ $21^\circ 31' 54.16''$ $16^\circ 5' 10.68''$	30.52251	See page 48.
175. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=7.5278 \\ c=6.1874, & z=4.8370 \end{cases}$	$36^\circ 12' 7.81''$ $26^\circ 28' 18.40''$ $15^\circ 27' 54.53''$	$42^\circ 19' 33.89''$ $29^\circ 32' 53.45''$ $20^\circ 29' 44.11''$	$44^\circ 59' 12.02''$ $30^\circ 33' 22.03''$ $21^\circ 40' 30.73''$	$43^\circ 1' 57.49''$ $32^\circ 1' 21.46''$ $17^\circ 7' 4.08''$	$62^\circ 42' 44.09''$ $23^\circ 37' 45.15''$ $17^\circ 4' 40.45''$	$60^\circ 45' 43.98''$ $22^\circ 45' 34.06''$ $13^\circ 40' 42.50''$	27.39224	
176. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=4.8370 \\ c=6.1874, & z=7.5278 \end{cases}$	$36^\circ 12' 7.81''$ $13^\circ 40' 10.53''$ $25^\circ 8' 46.85''$	$42^\circ 19' 33.89''$ $27^\circ 48' 15.68''$ $17^\circ 59' 20.22''$	$37^\circ 2' 57.47''$ $17^\circ 7' 4.08''$ $21^\circ 40' 30.73''$	$58^\circ 20' 29.25''$ $32^\circ 1' 21.46''$ $30^\circ 33' 22.03''$	$73^\circ 30' 56.48''$ $16^\circ 35' 54.07''$ $11^\circ 28' 21.69''$	$69^\circ 19' 21.06''$ $9^\circ 16' 52.08''$ $16^\circ 6' 46.13''$	18.29262	
177. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=6.1874 \\ c=4.8370, & z=7.5278 \end{cases}$	$39^\circ 20' 42.98''$ $17^\circ 59' 20.22''$ $25^\circ 8' 46.85''$	$38^\circ 16' 21.90''$ $27^\circ 48' 15.68''$ $13^\circ 40' 10.53''$	$37^\circ 2' 57.47''$ $22^\circ 19' 58.79''$ $16^\circ 49' 25.87''$	$58^\circ 20' 29.25''$ $33^\circ 49' 51.15''$ $29^\circ 23' 39.31''$	$72^\circ 22' 25.23''$ $18^\circ 20' 43.82''$ $9^\circ 42' 40.63''$	$67^\circ 37' 3.92''$ $12^\circ 28' 36.85''$ $16^\circ 46' 6.85''$	19.49619	
178. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=7.5278 \\ c=4.8370, & z=6.1874 \end{cases}$	$39^\circ 20' 42.98''$ $26^\circ 18' 8.81''$ $20^\circ 16' 30.92''$	$38^\circ 16' 21.90''$ $29^\circ 1' 10.18''$ $15^\circ 20' 53.15''$	$40^\circ 42' 18.90''$ $29^\circ 23' 39.31''$ $16^\circ 49' 25.87''$	$48^\circ 20' 58.04''$ $33^\circ 49' 51.15''$ $22^\circ 19' 58.79''$	$63^\circ 15' 39.81''$ $24^\circ 13' 38.13''$ $13^\circ 22' 56.58''$	$60^\circ 27' 23.83''$ $22^\circ 1' 4.33''$ $17^\circ 20' 1.30''$	27.14508	
179. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=4.8370 \\ c=7.5278, & z=6.1874 \end{cases}$	$33^\circ 13' 13.75''$ $15^\circ 20' 53.15''$ $20^\circ 16' 30.92''$	$50^\circ 35' 7.77''$ $29^\circ 1' 10.18''$ $26^\circ 18' 8.81''$	$40^\circ 42' 18.90''$ $16^\circ 49' 58.84''$ $26^\circ 51' 29.37''$	$48^\circ 20' 58.04''$ $29^\circ 55' 16.88''$ $22^\circ 34' 53.39''$	$70^\circ 5' 10.23''$ $16^\circ 56' 41.68''$ $15^\circ 44' 17.31''$	$69^\circ 55' 32.04''$ $10^\circ 31' 18.50''$ $13^\circ 36' 23.04''$	20.40044	
180. $\begin{cases} a=9.4106, & x=8.5602 \\ b=8.0746, & y=6.1874 \\ c=7.5278, & z=4.8370 \end{cases}$	$33^\circ 13' 13.75''$ $20^\circ 29' 44.11''$ $15^\circ 27' 54.53''$	$50^\circ 35' 7.77''$ $29^\circ 32' 53.45''$ $26^\circ 28' 18.40''$	$44^\circ 59' 12.02''$ $22^\circ 34' 53.39''$ $26^\circ 51' 29.37''$	$43^\circ 1' 57.49''$ $29^\circ 55' 16.88''$ $16^\circ 49' 58.84''$	$68^\circ 24' 28.22''$ $18^\circ 11' 22.58''$ $16^\circ 44' 24.56''$	$68^\circ 39' 0.30''$ $14^\circ 30' 55.65''$ $11^\circ 10' 30.20''$	21.79769	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}X$, $\frac{1}{2}Y$, $\frac{1}{2}Z$	VOLUME.	REMARKS.
181. $\begin{cases} a=9.4826, & x=7.5998 \\ b=8.5612, & y=8.0736 \\ c=6.1874, & z=4.8370 \end{cases}$	$29^{\circ}44'2.48''$ $28^{\circ}45'30.22''$ $15^{\circ}16'41.32''$	$33^{\circ}22'42.13''$ $32^{\circ}3'59.67''$ $20^{\circ}8'17.30''$	$41^{\circ}6'12.48''$ $38^{\circ}0'5.73''$ $17^{\circ}53'40.67''$	$48^{\circ}3'55.23''$ $38^{\circ}25'18.81''$ $22^{\circ}11'50.65''$	$41^{\circ}57'5.90''$ $38^{\circ}25'39.51''$ $17^{\circ}58'5.03''$	35.69917	In these three tetra- hedra $a-x=1.88:8$ $b+y=16.634:8$ $c=61874, z=4.8370$ and (181) is the one of maximum volume under these condi- tions, having $A+\lambda$ $=180^{\circ}$, $B-Y=0^{\circ}$.
182. $\begin{cases} a=0.482093, & x=7.599293 \\ b=8.561929, & y=8.072871 \\ c=6.1874, & z=4.8370 \end{cases}$	$29^{\circ}43'44.87''$ $28^{\circ}45'24.43''$ $15^{\circ}16'43.26''$	$33^{\circ}22'43.98''$ $32^{\circ}4'24.61''$ $20^{\circ}8'22.12''$	$41^{\circ}6'13.45''$ $38^{\circ}0'26.82''$ $17^{\circ}53'46.40''$	$48^{\circ}3'24.77''$ $38^{\circ}25'54.77''$ $22^{\circ}11'52.59''$	$41^{\circ}56'38.85''$ $38^{\circ}25'56.97''$ $17^{\circ}58'12.84''$	35.69918	
183. $\begin{cases} a=9.482061, & x=7.599261 \\ b=8.561973, & y=8.072827 \\ c=6.1874, & z=4.8370 \end{cases}$	$29^{\circ}43'43.78''$ $28^{\circ}45'24.43''$ $15^{\circ}16'43.41''$	$33^{\circ}22'44.06''$ $32^{\circ}4'26.13''$ $20^{\circ}8'22.40''$	$41^{\circ}6'13.52''$ $38^{\circ}0'28.10''$ $17^{\circ}53'46.74''$	$48^{\circ}3'22.86''$ $38^{\circ}25'56.97''$ $22^{\circ}11'52.71''$	$41^{\circ}56'37.14''$ $38^{\circ}25'56.97''$ $17^{\circ}57'46.64''$	35.69918	
184. $\begin{cases} a=5.239019, & x=5.239909 \\ b=3.868676, & y=3.900688 \\ c=3.506886, & z=5.442099 \end{cases}$	$32^{\circ}45'54.94''$ $21^{\circ}14'18.09''$ $35^{\circ}59'46.97''$	$32^{\circ}50'15.80''$ $20^{\circ}51'47.42''$ $36^{\circ}12'56.78''$	$32^{\circ}32'45.13''$ $20^{\circ}42'36.09''$ $35^{\circ}41'14.78''$	$36^{\circ}34'38.90''$ $22^{\circ}8'18.11''$ $45^{\circ}10'10.26''$	$36^{\circ}34'22.10''$ $22^{\circ}37'11.50''$ $44^{\circ}49'49.74''$	11.25615	$\sigma_1=\sigma_2=90^{\circ}$, $C+Z=180^{\circ}$, $\sigma_1-\beta_1=\sigma_2-\beta_2$, $\sigma_1-\gamma_1=\sigma_2-\gamma_2$ $\sigma_2-\gamma_2=\sigma_1-\gamma_1$
185. $\begin{cases} a=4.600606, & x=4.705 \\ b=3.763106, & y=4.581051 \\ c=5.391406, & z=4.616923 \end{cases}$	$29^{\circ}20'28.02''$ $27^{\circ}35'36.68''$ $33^{\circ}3'55.30''$	$30^{\circ}23'49.33''$ $24^{\circ}5'41.43''$ $35^{\circ}30'29.24''$	$26^{\circ}53'54.08''$ $21^{\circ}33'0.30''$ $29^{\circ}27'52.86''$	$34^{\circ}17'7.52''$ $26^{\circ}0'36.21''$ $45^{\circ}0'0.00''$	$33^{\circ}41'24.26''$ $31^{\circ}15'24.44''$ $40^{\circ}34'30.18''$	10.78158	$\sigma_1=\sigma_2=90^{\circ}$, $C=90^{\circ}$, $\sigma_1-\beta_1=\sigma_2-\beta_2$, $\sigma_1-\gamma_1=\sigma_2-\gamma_2$ $\sigma_2-\gamma_2=\sigma_1-\gamma_1$
186. $\begin{cases} a=13.2480, & x=6.0561 \\ b=11.5857, & y=11.4048 \\ c=9.7425, & z=9.5616 \end{cases}$	$15^{\circ}45'11.24''$ $28^{\circ}31'52.05''$ $22^{\circ}20'4.42''$	$16^{\circ}2'3.76''$ $29^{\circ}12'20.80''$ $22^{\circ}54'3.71''$	$38^{\circ}34'4.24''$ $45^{\circ}38'23.82''$ $28^{\circ}28'42.82''$	$36^{\circ}51'50.22''$ $50^{\circ}3'54.23''$ $27^{\circ}58'24.41''$	$18^{\circ}55'26.67''$ $49^{\circ}54'22.30''$ $27^{\circ}48'52.48''$	90.22281	A group of six tetra- hedra, all having one common pair of op- posite edges; and the other four edges having the same lengths in each but differently disposed.
187. $\begin{cases} a=13.2480, & x=9.5616 \\ b=11.5857, & y=11.4048 \\ c=9.7425, & z=6.0561 \end{cases}$	$26^{\circ}12'10.45''$ $28^{\circ}31'52.05''$ $13^{\circ}35'35.53''$	$28^{\circ}28'42.82''$ $30^{\circ}27'59.69''$ $22^{\circ}54'3.71''$	$38^{\circ}34'4.24''$ $45^{\circ}38'23.82''$ $16^{\circ}2'3.76''$	$45^{\circ}48'31.02''$ $39^{\circ}56'5.78''$ $26^{\circ}55'26.49''$	$35^{\circ}22'33.80''$ $44^{\circ}54'58.08''$ $16^{\circ}29'29.27''$	90.22281	The volumes are equal two and two; and in any pair of equal volume, the sum of two corre- sponding dihedral angles is supplement- ary. Several rela- tions of the form $A \pm X = B \pm Y$ hold.
188. $\begin{cases} a=13.2480, & x=6.0561 \\ b=11.5857, & y=11.4048 \\ c=9.5616, & z=9.7425 \end{cases}$	$15^{\circ}43'58.45''$ $28^{\circ}30'1.20''$ $22^{\circ}47'39.93''$	$16^{\circ}2'7.71''$ $29^{\circ}4'39.86''$ $22^{\circ}26'8.00''$	$38^{\circ}54'50.80''$ $46^{\circ}30'59.55''$ $29^{\circ}20'3.57''$	$36^{\circ}53'11.68''$ $50^{\circ}36'46.33''$ $27^{\circ}16'31.85''$	$18^{\circ}54'24.01''$ $49^{\circ}22'14.34''$ $28^{\circ}31'3.84''$	90.14519	
189. $\begin{cases} a=13.2480, & x=9.7425 \\ b=11.5857, & y=11.4048 \\ c=9.5616, & z=6.0561 \end{cases}$	$26^{\circ}55'6.71''$ $28^{\circ}30'1.20''$ $13^{\circ}35'35.53''$	$29^{\circ}20'3.57''$ $30^{\circ}27'59.69''$ $22^{\circ}26'8.00''$	$38^{\circ}54'50.80''$ $46^{\circ}30'59.55''$ $16^{\circ}2'7.71''$	$46^{\circ}28'18.81''$ $36^{\circ}52'42.84''$ $26^{\circ}12'10.45''$	$36^{\circ}43'25.62''$ $43^{\circ}58'30.73''$ $16^{\circ}25'32.64''$	90.14519	
190. $\begin{cases} a=13.2480, & x=9.5616 \\ b=11.5857, & y=11.4048 \\ c=6.0561, & z=9.7425 \end{cases}$	$27^{\circ}45'2.00''$ $29^{\circ}36'35.36''$ $22^{\circ}47'39.93''$	$26^{\circ}31'41.83''$ $29^{\circ}4'39.86''$ $13^{\circ}34'17.34''$	$46^{\circ}49'7.30''$ $46^{\circ}30'59.55''$ $27^{\circ}15'41.69''$	$46^{\circ}9'12.36''$ $45^{\circ}34'2.26''$ $16^{\circ}14'16.27''$	$35^{\circ}4'52.14''$ $39^{\circ}20'54.39''$ $27^{\circ}18'36.49''$	89.43556	

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
191. $\begin{cases} a=13.2480, & x=9.7425 \\ b=11.5857, & y=11.4048 \\ c=6.0561, & z=9.5616 \end{cases}$	$28^\circ 36' 24.94''$ $29^\circ 36' 35.36''$ $22^\circ 20' 4.42''$	$27^\circ 15' 41.60''$ $29^\circ 12' 20.80''$ $13^\circ 34' 17.34''$	$38^\circ 27' 34.78''$ $36^\circ 12' 36.48''$ $15^\circ 45' 11.24''$	$46^\circ 49' 7.30''$ $45^\circ 38' 23.82''$ $26^\circ 31' 41.83''$	$46^\circ 47' 40.05''$ $44^\circ 25' 57.77''$ $16^\circ 11' 15.28''$	$36^\circ 27' 6.17''$ $38^\circ 48' 48.45''$ $26^\circ 31' 49.16''$	89.43556	See previous page.
192. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=3.5616 \\ c=3.1189, & z=2.8560 \end{cases}$	$26^\circ 56' 50.38''$ $28^\circ 6' 14.81''$ $20^\circ 45' 52.88''$	$29^\circ 23' 20.47''$ $31^\circ 18' 4.55''$ $23^\circ 20' 58.78''$	$37^\circ 56' 2.57''$ $35^\circ 49' 42.50''$ $25^\circ 52' 27.07''$	$38^\circ 32' 46.41''$ $37^\circ 10' 42.55''$ $24^\circ 46' 57.03''$	$43^\circ 6' 11.78''$ $40^\circ 9' 33.98''$ $26^\circ 57' 7.56''$	$35^\circ 51' 19.78''$ $38^\circ 47' 57.58''$ $25^\circ 35' 31.16''$	4.344699	
193. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=2.8560 \\ c=3.1189, & z=3.5616 \end{cases}$	$26^\circ 56' 50.38''$ $21^\circ 33' 30.82''$ $26^\circ 19' 2.30''$	$29^\circ 23' 20.47''$ $29^\circ 17' 39.70''$ $24^\circ 8' 24.41''$	$34^\circ 23' 18.00''$ $24^\circ 46' 57.03''$ $25^\circ 52' 27.07''$	$44^\circ 18' 4.77''$ $37^\circ 10' 42.55''$ $35^\circ 49' 42.50''$	$46^\circ 53' 48.22''$ $36^\circ 59' 13.17''$ $28^\circ 36' 4.57''$	$37^\circ 11' 24.76''$ $27^\circ 16' 49.71''$ $35^\circ 39' 58.31''$	4.344699	
194. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=3.1189 \\ c=3.5616, & z=2.8560 \end{cases}$	$25^\circ 40' 26.75''$ $23^\circ 20' 58.78''$ $20^\circ 45' 52.88''$	$32^\circ 23' 43.67''$ $31^\circ 18' 4.55''$ $28^\circ 6' 14.81''$	$33^\circ 56' 2.57''$ $30^\circ 47' 8.57''$ $30^\circ 2' 44.17''$	$38^\circ 32' 46.41''$ $34^\circ 16' 49.08''$ $26^\circ 49' 7.76''$	$44^\circ 44' 42.67''$ $35^\circ 21' 56.22''$ $31^\circ 40' 46.96''$	$38^\circ 20' 24.06''$ $32^\circ 32' 5.25''$ $27^\circ 51' 5.15''$	4.452748	Another system of six tetrahedra, having similar relations to those stated for the set (186) to (191).
195. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=2.8560 \\ c=3.5616, & z=3.1189 \end{cases}$	$25^\circ 40' 26.75''$ $21^\circ 11' 35.30''$ $22^\circ 48' 30.09''$	$32^\circ 23' 43.67''$ $30^\circ 36' 11.57''$ $28^\circ 36' 20.47''$	$36^\circ 35' 18.34''$ $26^\circ 49' 7.76''$ $30^\circ 2' 44.17''$	$40^\circ 12' 4.28''$ $34^\circ 16' 49.08''$ $36^\circ 47' 8.57''$	$45^\circ 15' 17.33''$ $35^\circ 21' 56.22''$ $32^\circ 26' 53.18''$	$38^\circ 31' 32.32''$ $28^\circ 38' 11.21''$ $31^\circ 33' 14.25''$	4.452748	
196. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=3.5616 \\ c=2.8560, & z=3.1189 \end{cases}$	$27^\circ 34' 7.34''$ $28^\circ 36' 20.47''$ $22^\circ 48' 30.09''$	$28^\circ 28' 30.67''$ $30^\circ 36' 11.57''$ $21^\circ 11' 35.30''$	$36^\circ 35' 18.34''$ $23^\circ 27' 22.35''$ $23^\circ 27' 22.35''$	$40^\circ 12' 4.28''$ $38^\circ 58' 30.31''$ $37^\circ 16' 46.54''$	$43^\circ 21' 24.63''$ $40^\circ 59' 23.25''$ $24^\circ 48' 27.08''$	$35^\circ 38' 23.94''$ $38^\circ 0' 25.32''$ $27^\circ 47' 25.01''$	4.327226	
197. $\begin{cases} a=4.1773, & x=3.2088 \\ b=3.8245, & y=3.1189 \\ c=2.8560, & z=3.5616 \end{cases}$	$27^\circ 34' 7.34''$ $24^\circ 8' 24.41''$ $26^\circ 19' 2.30''$	$28^\circ 28' 30.67''$ $29^\circ 17' 39.70''$ $31^\circ 33' 30.82''$	$34^\circ 23' 18.00''$ $21^\circ 16' 46.54''$ $23^\circ 27' 22.35''$	$44^\circ 18' 4.77''$ $38^\circ 58' 30.31''$ $34^\circ 14' 42.79''$	$46^\circ 38' 35.38''$ $38^\circ 47' 48.05''$ $25^\circ 43' 45.48''$	$36^\circ 47' 17.47''$ $30^\circ 22' 42.41''$ $34^\circ 8' 51.72''$	4.327226	
198. $\begin{cases} a=7.39443, & x=6.5208 \\ b=6.52081, & y=7.39442 \\ c=4.77357, & z=5.64718 \end{cases}$	$34^\circ 15' 52.46''$ $35^\circ 35' 3.59''$ $23^\circ 41' 11.82''$	$29^\circ 5' 4.17''$ $29^\circ 5' 4.24''$ $18^\circ 49' 52.42''$	$37^\circ 13' 43.94''$ $31^\circ 13' 43.87''$ $21^\circ 28' 14.68''$	$35^\circ 35' 3.99''$ $34^\circ 15' 52.86''$ $23^\circ 41' 11.96''$	$41^\circ 51' 35.82''$ $38^\circ 53' 19.77''$ $28^\circ 19' 43.09''$	$38^\circ 53' 19.58''$ $41^\circ 51' 35.63''$ $25^\circ 17' 58.95''$	26.10944	Shortest distance between c and z = 5.840578.
199. $\begin{cases} a=7.39443, & x=6.5208 \\ b=4.77357, & y=7.39442 \\ c=6.52081, & z=5.64718 \end{cases}$	$34^\circ 15' 52.46''$ $31^\circ 55' 6.33''$ $24^\circ 53' 45.80''$	$29^\circ 5' 4.17''$ $20^\circ 6' 14.20''$ $26^\circ 9' 47.06''$	$45^\circ 0' 0.00''$ $37^\circ 13' 43.87''$ $34^\circ 15' 52.86''$	$31^\circ 55' 6.61''$ $21^\circ 28' 14.68''$ $23^\circ 41' 11.96''$	$48^\circ 8' 24.22''$ $24^\circ 29' 21.19''$ $33^\circ 50' 8.67''$	$41^\circ 35' 26.71''$ $36^\circ 37' 58.64''$ $27^\circ 17' 11.16''$	26.10944	
200. $\begin{cases} a=7.39443, & x=6.5208 \\ b=6.52081, & y=4.77357 \\ c=7.39442, & z=5.64718 \end{cases}$	$27^\circ 43' 47.22''$ $18^\circ 49' 52.42''$ $23^\circ 41' 11.82''$	$38^\circ 29' 3.70''$ $29^\circ 5' 4.24''$ $35^\circ 35' 3.59''$	$37^\circ 13' 43.94''$ $22^\circ 44' 52.63''$ $34^\circ 33' 25.33''$	$35^\circ 35' 3.99''$ $28^\circ 46' 3.67''$ $23^\circ 41' 11.96''$	$44^\circ 14' 24.21''$ $30^\circ 13' 46.74''$ $37^\circ 52' 15.53''$	$43^\circ 1' 49.46''$ $23^\circ 51' 57.98''$ $31^\circ 30' 26.77''$	26.65186	See next page.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}X$, $\frac{1}{2}Y$, $\frac{1}{2}Z$	VOLUME.	REMARKS.
201. $\left\{ \begin{array}{l} a=7.39403, \\ b=6.52081, \\ c=7.39442, \end{array} \right. \begin{array}{l} x=6.5208 \\ y=5.54718 \\ z=4.77357 \end{array}$	$27^{\circ}43'47.22''$ $22^{\circ}26'54.85''$ $19^{\circ}45'0.50''$	$38^{\circ}29'3.70''$ $30^{\circ}9'56.54''$ $33^{\circ}46'32.41''$	$43^{\circ}41'54.57''$ $29^{\circ}26'6.21''$ $25^{\circ}14'24.22''$	26.65187	Another system of six tetrahedra, in all of which $a = p + 3q$, $x = 4p$, the other four lengths of edges being $p + q$, $p^2 - 3q^2$, $4pq + 9q^2$, $4p^2 - 9q^2$, $p = \frac{x}{4}$, $q = \frac{y}{12}$, 209 $\frac{1}{2}$
202. $\left\{ \begin{array}{l} a=7.39443, \\ b=6.52081, \\ c=5.04718, \end{array} \right. \begin{array}{l} x=6.5208 \\ y=4.77357 \\ z=7.39442 \end{array}$	$32^{\circ}10'13.83''$ $20^{\circ}6'14.20''$ $31^{\circ}55'6.33''$	$30^{\circ}9'56.49''$ $26^{\circ}9'47.06''$ $24^{\circ}53'45.80''$	$40^{\circ}13'4.22''$ $22^{\circ}27'52.63''$ $39^{\circ}32'24.30''$	25.9299	$p = \frac{x}{4}$, $q = \frac{y}{12}$, 209 $\frac{1}{2}$
203. $\left\{ \begin{array}{l} a=7.39443, \\ b=6.52081, \\ c=5.04718, \end{array} \right. \begin{array}{l} x=6.5208 \\ y=7.39442 \\ z=4.77357 \end{array}$	$32^{\circ}10'13.83''$ $33^{\circ}46'32.41''$ $19^{\circ}45'0.50''$	$30^{\circ}9'56.49''$ $30^{\circ}9'56.54''$ $22^{\circ}26'54.85''$	$40^{\circ}13'4.22''$ $22^{\circ}27'52.63''$ $39^{\circ}32'24.30''$	25.9299	$p = \frac{x}{4}$, $q = \frac{y}{12}$, 209 $\frac{1}{2}$
204. $\left\{ \begin{array}{l} a=3.4677, \\ b=3.2355, \\ c=4.0137, \end{array} \right. \begin{array}{l} x=2.8788 \\ y=3.1110 \\ z=3.4248 \end{array}$	$22^{\circ}36'58.52''$ $24^{\circ}14'4.04''$ $30^{\circ}40'1.93''$	$25^{\circ}59'54.52''$ $27^{\circ}59'41.20''$ $37^{\circ}29'14.52''$	$26^{\circ}59'41.23''$ $29^{\circ}15'54.92''$ $44^{\circ}18'41.23''$	3.987851	In all the four tetrahedra, (204) to (207), $a = 3.4677$, $x = 2.8788$, $b + y = a + x$, $c - z = a - x$; and (207) is the tetrahedron of max. volume under these conditions. Observe that in passing from (206) to (207) $B - Y$ is unaltered by the changes in c , z .
205. $\left\{ \begin{array}{l} a=3.4677, \\ b=3.2421, \\ c=3.9489, \end{array} \right. \begin{array}{l} x=2.8788 \\ y=3.1044 \\ z=3.3600 \end{array}$	$22^{\circ}57'9.14''$ $24^{\circ}28'50.94''$ $29^{\circ}59'39.19''$	$26^{\circ}21'15.27''$ $28^{\circ}20'9.78''$ $36^{\circ}48'57.18''$	$27^{\circ}52'56.49''$ $30^{\circ}7'44.76''$ $42^{\circ}42'26.60''$	3.995449	
206. $\left\{ \begin{array}{l} a=3.4677, \\ b=3.242074, \\ c=3.95078, \end{array} \right. \begin{array}{l} x=2.8788 \\ y=3.104426 \\ z=3.36188 \end{array}$	$22^{\circ}56'33.59''$ $24^{\circ}28'20.38''$ $30^{\circ}0'46.56''$	$26^{\circ}20'40.36''$ $28^{\circ}19'39.91''$ $36^{\circ}50'5.77''$	$27^{\circ}51'27.92''$ $30^{\circ}6'13.00''$ $42^{\circ}45'15.01''$	3.995455	
207. $\left\{ \begin{array}{l} a=3.4677, \\ b=3.242074, \\ c=3.95083, \end{array} \right. \begin{array}{l} x=2.8788 \\ y=3.104426 \\ z=3.36193 \end{array}$	$22^{\circ}56'32.66''$ $24^{\circ}28'19.53''$ $30^{\circ}0'48.32''$	$26^{\circ}20'39.46''$ $28^{\circ}19'39.15''$ $36^{\circ}50'7.72''$	$27^{\circ}51'27.92''$ $30^{\circ}6'13.00''$ $42^{\circ}45'15.01''$	3.995456	
208. $\left\{ \begin{array}{l} a=0.807817, \\ b=8.9763, \\ c=8.131859, \end{array} \right. \begin{array}{l} x=5.58317 \\ y=4.7268 \\ z=5.571241 \end{array}$	$18^{\circ}45'16.13''$ $14^{\circ}19'35.39''$ $17^{\circ}4'16.09''$	$32^{\circ}21'47.08''$ $32^{\circ}21'47.08''$ $27^{\circ}47'25.68''$	$37^{\circ}47'45.05''$ $24^{\circ}51'11.61''$ $31^{\circ}36'43.00''$	31.10119	$A - X = B - Y$, $C + Z = B + Y$, $B - X = 75^{\circ}35'30.10''$, $S_1 = 91^{\circ}120'346.54''$
209. $\left\{ \begin{array}{l} a=5.8341, \\ b=5.2931, \\ c=6.3758, \end{array} \right. \begin{array}{l} x=2.9448 \\ y=3.4858 \\ z=3.4865 \end{array}$	$13^{\circ}37'59.26''$ $16^{\circ}23'50.93''$ $18^{\circ}3'12.12''$	$24^{\circ}59'0.56''$ $31^{\circ}43'53.01''$ $41^{\circ}5'17.14''$	$24^{\circ}27'35.87''$ $31^{\circ}30'42.88''$ $37^{\circ}55'8.72''$	7.895052	$A + X = B + Y$, $A - X = C - Z$, $A = Z$
210. $\left\{ \begin{array}{l} a=7.2841, \\ b=6.736393, \\ c=7.860554, \end{array} \right. \begin{array}{l} x=2.9448 \\ y=3.49497 \\ z=3.521254 \end{array}$	$10^{\circ}46'36.21''$ $13^{\circ}9'58.42''$ $14^{\circ}22'37.19''$	$24^{\circ}48'56.15''$ $33^{\circ}27'32.14''$ $43^{\circ}2'16.15''$	$23^{\circ}22'54.06''$ $31^{\circ}23'17.35''$ $37^{\circ}19'37.71''$	10.09838	$A + X = B + Y$, $A - X = C - Z$, $B = Y$, $C + Z = 180^{\circ}$. Vol. a max. for $a, x, b + y, c - z$ given.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
211. $\begin{cases} a=7.71042, & x=5.618158 \\ b=8.5445, & y=6.6434 \\ c=7.6147, & z=5.7130 \end{cases}$	$20^\circ 5' 13.98''$ $25^\circ 41' 16.43''$ $20^\circ 22' 37.42''$	$26^\circ 43' 18.50''$ $38^\circ 44' 43.74''$ $31^\circ 47' 6.53''$	$30^\circ 52' 38.84''$ $35^\circ 53' 22.25''$ $30^\circ 29' 7.68''$	$32^\circ 31' 37.04''$ $39^\circ 25' 38.34''$ $27^\circ 23' 19.35''$	$33^\circ 36' 44.42''$ $46^\circ 56' 1.00''$ $33^\circ 6' 52.90''$	$27^\circ 35' 20.32''$ $41^\circ 54' 19.94''$ $28^\circ 5' 11.84''$	35.00578	$A + X = C + Z,$ $B - Y = C - Z,$ $\sigma_2 = \sigma_3 = 97^\circ 15' 8.77''$
212. $\begin{cases} a=9.31534, & x=5.06584 \\ b=8.9763, & y=4.7268 \\ c=8.01066, & z=5.69244 \end{cases}$	$17^\circ 3' 4.15''$ $15^\circ 14' 45.59''$ $18^\circ 6' 6.27''$	$28^\circ 38' 34.36''$ $34^\circ 19' 35.22''$ $29^\circ 39' 17.25''$	$37^\circ 34' 18.51''$ $25^\circ 51' 48.19''$ $31^\circ 13' 27.55''$	$45^\circ 5' 57.16''$ $41^\circ 43' 28.30''$ $35^\circ 29' 37.45''$	$46^\circ 14' 22.54''$ $41^\circ 30' 0.62''$ $33^\circ 12' 9.72''$	$31^\circ 51' 31.23''$ $27^\circ 7' 9.31''$ $35^\circ 25' 0.21''$	30	$A - X = B - Y,$ $B + Y = C + Z$
213. $\begin{cases} a=9.8933, & x=5.6438 \\ b=8.9763, & y=4.7268 \\ c=8.0107, & z=5.6924 \end{cases}$	$19^\circ 8' 23.15''$ $14^\circ 5' 38.70''$ $17^\circ 20' 32.92''$	$32^\circ 24' 40.59''$ $31^\circ 54' 21.84''$ $26^\circ 35' 27.42''$	$40^\circ 45' 5.24''$ $24^\circ 38' 31.14''$ $30^\circ 46' 45.40''$	$49^\circ 18' 53.88''$ $40^\circ 4' 51.45''$ $32^\circ 56' 48.27''$	$52^\circ 4' 27.50''$ $41^\circ 30' 0.62''$ $33^\circ 12' 9.72''$	$38^\circ 31' 35.02''$ $33^\circ 59' 55.59''$ $31^\circ 46' 9.25''$	31.06101	$A - X = B - Y,$ $B + Y = C + Z$
214. $\begin{cases} a=9.8671, & x=5.6176 \\ b=8.9763, & y=4.7268 \\ c=8.1807, & z=5.5224 \end{cases}$	$18^\circ 56' 0.90''$ $14^\circ 13' 32.89''$ $16^\circ 49' 52.50''$	$32^\circ 58' 16.63''$ $32^\circ 8' 36.50''$ $27^\circ 46' 14.72''$	$41^\circ 1' 31.00''$ $25^\circ 6' 8.84''$ $31^\circ 41' 1.90''$	$48^\circ 0' 12.39''$ $39^\circ 22' 57.20''$ $31^\circ 55' 34.53''$	$51^\circ 14' 52.86''$ $37^\circ 10' 15.09''$ $30^\circ 53' 36.10''$	$38^\circ 41' 44.11''$ $24^\circ 37' 6.40''$ $30^\circ 53' 45.39''$	31.12113	$A + X = 186^\circ, C = Z,$ in all four; in (216) so that this is the tetrahedron of maxi- mum volume under the conditions.
215. $\begin{cases} a=9.86936, & x=5.61986 \\ b=8.9763, & y=4.7268 \\ c=8.18092, & z=5.52218 \end{cases}$	$18^\circ 56' 29.39''$ $14^\circ 13' 16.98''$ $16^\circ 49' 38.57''$	$32^\circ 59' 15.00''$ $32^\circ 8' 1.60''$ $27^\circ 45' 35.96''$	$41^\circ 2' 10.83''$ $25^\circ 5' 50.30''$ $31^\circ 40' 57.77''$	$48^\circ 1' 8.99''$ $39^\circ 22' 32.84''$ $31^\circ 54' 54.70''$	$51^\circ 16' 21.93''$ $37^\circ 9' 7.90''$ $30^\circ 52' 42.75''$	$38^\circ 43' 31.60''$ $24^\circ 30' 17.66''$ $30^\circ 52' 42.81''$	31.12115	$a - x = b - y = 4.2495,$ $c + z = b + y = 13.7031.$ $\sigma_4 - \sigma_3$ is very small ($0^\circ 0' 23''$).
216. $\begin{cases} a=9.869433, & x=5.619933 \\ b=8.9763, & y=4.7268 \\ c=8.180921, & z=5.522179 \end{cases}$	$18^\circ 56' 30.38''$ $14^\circ 13' 16.44''$ $16^\circ 49' 38.22''$	$32^\circ 59' 16.77''$ $32^\circ 8' 0.44''$ $27^\circ 45' 34.57''$	$41^\circ 2' 21.34''$ $25^\circ 5' 49.66''$ $31^\circ 40' 57.55''$	$48^\circ 1' 8.99''$ $39^\circ 22' 32.84''$ $31^\circ 54' 53.57''$	$51^\circ 16' 24.88''$ $37^\circ 9' 7.90''$ $30^\circ 52' 40.93''$	$38^\circ 43' 35.12''$ $24^\circ 30' 16.05''$ $30^\circ 52' 40.93''$	31.12115	
217. $\begin{cases} a=9.0481, & x=8.8901 \\ b=8.9217, & y=9.0797 \\ c=6.4025, & z=6.3040 \end{cases}$	$34^\circ 20' 5.56''$ $34^\circ 47' 28.38''$ $20^\circ 31' 59.27''$	$33^\circ 55' 43.65''$ $34^\circ 11' 43.75''$ $20^\circ 40' 55.67''$	$35^\circ 16' 16.98''$ $35^\circ 32' 34.11''$ $21^\circ 3' 58.89''$	$34^\circ 31' 35.95''$ $34^\circ 35' 55.55''$ $20^\circ 31' 42.24''$	$40^\circ 47' 59.34''$ $40^\circ 59' 25.55''$ $22^\circ 22' 6.00''$	$40^\circ 17' 31.62''$ $41^\circ 29' 53.27''$ $22^\circ 2' 35.08''$	52.31824	$V_1 = V_2 = V_3$ $A_1 + B_1 = 180^\circ$ $H_1 + R_1 = 180^\circ$ $B_2 - A_2 = 0^\circ$
218. $\begin{cases} a=9.0481, & x=8.8901 \\ b=8.9217, & y=9.0797 \\ c=6.4025, & z=6.3040 \end{cases}$	$34^\circ 20' 5.56''$ $22^\circ 4' 46.20''$ $30^\circ 20' 56.00''$	$33^\circ 55' 43.65''$ $29^\circ 28' 55.97''$ $22^\circ 31' 2.78''$	$39^\circ 10' 14.03''$ $30^\circ 31' 42.24''$ $21^\circ 3' 58.89''$	$45^\circ 24' 11.02''$ $34^\circ 35' 55.55''$ $35^\circ 32' 34.11''$	$49^\circ 12' 0.66''$ $33^\circ 49' 28.48''$ $24^\circ 53' 46.04''$	$44^\circ 24' 21.68''$ $24^\circ 11' 48.11''$ $34^\circ 41' 27.14''$	52.31824	
219. $\begin{cases} a=9.0481, & x=6.3040 \\ b=8.9217, & y=9.0797 \\ c=6.4025, & z=8.8901 \end{cases}$	$22^\circ 28' 34.72''$ $34^\circ 47' 28.38''$ $29^\circ 38' 50.74''$	$20^\circ 31' 42.24''$ $20^\circ 49' 17.24''$ $20^\circ 40' 55.67''$	$30^\circ 31' 46.02''$ $35^\circ 32' 34.11''$ $22^\circ 55' 36.30''$	$34^\circ 31' 35.95''$ $44^\circ 35' 48.98''$ $33^\circ 55' 43.05''$	$49^\circ 12' 0.66''$ $46^\circ 0' 34.44''$ $24^\circ 39' 35.45''$	$24^\circ 27' 25.49''$ $45^\circ 35' 16.57''$ $33^\circ 15' 57.76''$	52.31824	
220. $\begin{cases} a=9.0481, & x=8.8901 \\ b=8.9217, & y=9.0797 \\ c=6.3040, & z=6.4025 \end{cases}$	$34^\circ 30' 7.54''$ $34^\circ 57' 35.22''$ $20^\circ 52' 8.21''$	$33^\circ 46' 30.08''$ $34^\circ 2' 13.30''$ $20^\circ 20' 58.97''$	$35^\circ 5' 38.40''$ $35^\circ 21' 38.48''$ $20^\circ 43' 37.91''$	$34^\circ 41' 25.81''$ $34^\circ 46' 14.55''$ $20^\circ 51' 51.40''$	$40^\circ 47' 27.35''$ $40^\circ 59' 57.18''$ $22^\circ 3' 9.31''$	$40^\circ 18' 3.28''$ $41^\circ 20' 21.25''$ $22^\circ 21' 14.69''$	52.31891	See next page.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
221. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 34^{\circ}30', 7.54'' \\ 22^{\circ}31', 2.78'' \\ 30^{\circ}20'50.00'' \end{cases}$	$\begin{cases} 33^{\circ}46'30.08'' \\ 29^{\circ}28'55.97'' \\ 22^{\circ}46.20'' \end{cases}$	$\begin{cases} 30^{\circ}10'14.03'' \\ 20^{\circ}51'51.46'' \\ 20^{\circ}43'37.91'' \end{cases}$	$\begin{cases} 45^{\circ}24'11.02'' \\ 34^{\circ}46'14.55'' \\ 35^{\circ}21'38.46'' \end{cases}$	$\begin{cases} 49^{\circ}12'32.05'' \\ 33^{\circ}58'48.29'' \\ 24^{\circ}27'9.81'' \end{cases}$	$\begin{cases} 44^{\circ}25'26.33'' \\ 24^{\circ}37'51.40'' \\ 34^{\circ}31'16.72'' \end{cases}$	52.31891	$\begin{cases} V_1=V_2=V_3 \\ A_1+A_2=180^{\circ} \\ B_1+B_2=180^{\circ} \\ B_3-A_3=0 \end{cases}$
222. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 22^{\circ}55'36.30'' \\ 34^{\circ}57'35.22'' \\ 29^{\circ}49'17.24'' \end{cases}$	$\begin{cases} 20^{\circ}51'51.46'' \\ 29^{\circ}49'17.24'' \\ 22^{\circ}28'34.72'' \end{cases}$	$\begin{cases} 30^{\circ}10'14.03'' \\ 20^{\circ}51'51.46'' \\ 20^{\circ}43'37.91'' \end{cases}$	$\begin{cases} 45^{\circ}24'11.02'' \\ 34^{\circ}46'14.55'' \\ 35^{\circ}21'38.46'' \end{cases}$	$\begin{cases} 49^{\circ}12'32.05'' \\ 33^{\circ}58'48.29'' \\ 24^{\circ}27'9.81'' \end{cases}$	$\begin{cases} 44^{\circ}25'26.33'' \\ 24^{\circ}37'51.40'' \\ 34^{\circ}31'16.72'' \end{cases}$	52.31891	$\begin{cases} V_1=V_2=V_3 \\ A_1+A_2=180^{\circ} \\ B_1+B_2=180^{\circ} \\ B_3-A_3=0 \end{cases}$
223. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 29^{\circ}35'25.61'' \\ 20^{\circ}40'55.07'' \\ 20^{\circ}31'59.27'' \end{cases}$	$\begin{cases} 44^{\circ}23'49.76'' \\ 34^{\circ}11'43.75'' \\ 34^{\circ}47'28.38'' \end{cases}$	$\begin{cases} 35^{\circ}16'16.98'' \\ 23^{\circ}1'41.59'' \\ 30^{\circ}38'51.14'' \end{cases}$	$\begin{cases} 34^{\circ}31'35.92'' \\ 29^{\circ}45'43.22'' \\ 22^{\circ}34'28.65'' \end{cases}$	$\begin{cases} 45^{\circ}0'32.69'' \\ 33^{\circ}40'17.63'' \\ 34^{\circ}14'7.20'' \end{cases}$	$\begin{cases} 48^{\circ}32'28.77'' \\ 24^{\circ}53'34.21'' \\ 24^{\circ}41'35.01'' \end{cases}$	52.62805	$\begin{cases} V_1=V_2=V_3 \\ A_1+A_2=180^{\circ} \end{cases}$
224. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 29^{\circ}35'25.64'' \\ 20^{\circ}20'58.97'' \\ 20^{\circ}52'8.21'' \end{cases}$	$\begin{cases} 44^{\circ}23'49.76'' \\ 34^{\circ}2'13.30'' \\ 34^{\circ}57'35.22'' \end{cases}$	$\begin{cases} 35^{\circ}16'16.98'' \\ 23^{\circ}1'41.59'' \\ 30^{\circ}38'51.14'' \end{cases}$	$\begin{cases} 34^{\circ}31'35.92'' \\ 29^{\circ}45'43.22'' \\ 22^{\circ}34'28.65'' \end{cases}$	$\begin{cases} 45^{\circ}0'32.69'' \\ 33^{\circ}40'17.63'' \\ 34^{\circ}14'7.20'' \end{cases}$	$\begin{cases} 48^{\circ}32'28.77'' \\ 24^{\circ}53'34.21'' \\ 24^{\circ}41'35.01'' \end{cases}$	52.62805	$\begin{cases} V_1=V_2=V_3 \\ A_1+A_2=180^{\circ} \end{cases}$
225. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 21^{\circ}3'58.80'' \\ 30^{\circ}24'17.89'' \\ 20^{\circ}31'59.27'' \end{cases}$	$\begin{cases} 22^{\circ}24'51.21'' \\ 34^{\circ}11'43.75'' \\ 29^{\circ}22'3.15'' \end{cases}$	$\begin{cases} 35^{\circ}16'16.98'' \\ 45^{\circ}36'23.00'' \\ 34^{\circ}20'5.56'' \end{cases}$	$\begin{cases} 30^{\circ}13'38.96'' \\ 34^{\circ}35'55.55'' \\ 21^{\circ}58'45.79'' \end{cases}$	$\begin{cases} 34^{\circ}16'1.55'' \\ 44^{\circ}59'27.76'' \\ 33^{\circ}25'7.55'' \end{cases}$	$\begin{cases} 24^{\circ}30'22.96'' \\ 49^{\circ}39'46.69'' \\ 23^{\circ}58'27.89'' \end{cases}$	52.01483	$\begin{cases} V_1=V_2=V_3 \\ B_1+B_2=180^{\circ} \end{cases}$
226. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$	$\begin{cases} 20^{\circ}43'37.91'' \\ 30^{\circ}24'17.89'' \\ 20^{\circ}52'8.21'' \end{cases}$	$\begin{cases} 21^{\circ}58'45.79'' \\ 34^{\circ}2'13.30'' \\ 29^{\circ}22'3.15'' \end{cases}$	$\begin{cases} 35^{\circ}16'16.98'' \\ 45^{\circ}36'23.00'' \\ 34^{\circ}20'5.56'' \end{cases}$	$\begin{cases} 30^{\circ}13'38.96'' \\ 34^{\circ}35'55.55'' \\ 21^{\circ}58'45.79'' \end{cases}$	$\begin{cases} 34^{\circ}16'1.55'' \\ 44^{\circ}59'27.76'' \\ 33^{\circ}25'7.55'' \end{cases}$	$\begin{cases} 24^{\circ}30'22.96'' \\ 49^{\circ}39'46.69'' \\ 23^{\circ}58'27.89'' \end{cases}$	52.01483	$\begin{cases} V_1=V_2=V_3 \\ B_1+B_2=180^{\circ} \end{cases}$
227. $\begin{cases} a=9.952135, \\ b=9.128243, \\ c=8.365696, \end{cases}$	$\begin{cases} 16^{\circ}47'59.09'' \\ 12^{\circ}35'52.56'' \\ 15^{\circ}8'33.83'' \end{cases}$	$\begin{cases} 32^{\circ}53'44.95'' \\ 32^{\circ}53'44.95'' \\ 28^{\circ}6'35.66'' \end{cases}$	$\begin{cases} 41^{\circ}57'41.22'' \\ 24^{\circ}57'7.65'' \\ 32^{\circ}29'32.75'' \end{cases}$	$\begin{cases} 49^{\circ}17'31.84'' \\ 40^{\circ}42'28.16'' \\ 22^{\circ}24'51.21'' \end{cases}$	$\begin{cases} 51^{\circ}56'52.78'' \\ 38^{\circ}3'7.22'' \\ 31^{\circ}6'14.44'' \end{cases}$	$\begin{cases} 38^{\circ}3'7.22'' \\ 24^{\circ}9'21.66'' \\ 31^{\circ}6'14.44'' \end{cases}$	26.39783	$\begin{cases} A-X=B-Y, A+X=180^{\circ} \\ B+Y=C+Z, C=Z, B= \\ X: 5 \text{ equations, sufficient} \\ \text{to determine shape.} \end{cases}$
228. $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.39966, \end{cases}$	$\begin{cases} 44^{\circ}40'58.35'' \\ 45^{\circ}20'1.47'' \\ 49^{\circ}53'35.04'' \end{cases}$	$\begin{cases} 24^{\circ}18'24.67'' \\ 19^{\circ}9'44.14'' \\ 16^{\circ}29'42.74'' \end{cases}$	$\begin{cases} 20^{\circ}56'40.82'' \\ 26^{\circ}26'59.27'' \\ 17^{\circ}41'3.88'' \end{cases}$	$\begin{cases} 28^{\circ}10'15.79'' \\ 27^{\circ}37'57.77'' \\ 39^{\circ}14'36.66'' \end{cases}$	$\begin{cases} 28^{\circ}25'5.02'' \\ 27^{\circ}52'11.33'' \\ 23^{\circ}15'49.84'' \end{cases}$	$\begin{cases} 44^{\circ}44'8.67'' \\ 45^{\circ}17'2.35'' \\ 49^{\circ}53'23.84'' \end{cases}$	4.925029	$\begin{cases} A+X=B+Y=C+Z \\ S_1=S_2 \end{cases}$
229. $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.5555, \end{cases}$	$\begin{cases} 43^{\circ}25'51.50'' \\ 44^{\circ}7'59.92'' \\ 47^{\circ}58'6.25'' \end{cases}$	$\begin{cases} 24^{\circ}55'23.50'' \\ 20^{\circ}4'42.51'' \\ 17^{\circ}42'44.64'' \end{cases}$	$\begin{cases} 21^{\circ}58'10.24'' \\ 27^{\circ}10'16.22'' \\ 18^{\circ}59'44.35'' \end{cases}$	$\begin{cases} 28^{\circ}9'15.44'' \\ 27^{\circ}34'24.15'' \\ 37^{\circ}54'20.28'' \end{cases}$	$\begin{cases} 29^{\circ}17'39.60'' \\ 28^{\circ}39'42.84'' \\ 24^{\circ}36'13.62'' \end{cases}$	$\begin{cases} 43^{\circ}19'53.35'' \\ 43^{\circ}57'50.11'' \\ 48^{\circ}1'19.33'' \end{cases}$	5.28296	$\begin{cases} S_1 - S_2 = 12^{\circ}54.88'' \end{cases}$
230. $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.5575, \end{cases}$	$\begin{cases} 43^{\circ}24'55.28'' \\ 44^{\circ}7'6.12'' \\ 47^{\circ}56'39.00'' \end{cases}$	$\begin{cases} 24^{\circ}55'51.64'' \\ 20^{\circ}4'42.51'' \\ 17^{\circ}43'40.90'' \end{cases}$	$\begin{cases} 21^{\circ}58'10.24'' \\ 27^{\circ}10'16.22'' \\ 18^{\circ}59'44.35'' \end{cases}$	$\begin{cases} 28^{\circ}9'15.44'' \\ 27^{\circ}34'24.15'' \\ 37^{\circ}54'20.28'' \end{cases}$	$\begin{cases} 29^{\circ}17'39.60'' \\ 28^{\circ}39'42.84'' \\ 24^{\circ}36'13.62'' \end{cases}$	$\begin{cases} 43^{\circ}18'54.31'' \\ 43^{\circ}56'55.11'' \\ 47^{\circ}59'57.08'' \end{cases}$	5.287298	$\begin{cases} S_1 - S_2 = 12^{\circ}53.9'' \end{cases}$

EXAMPLES. X.—Continued.

	DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}X, \frac{1}{2}Y, \frac{1}{2}Z$	VOLUME.	REMARKS.
231.	$\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.5535, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2602 \end{cases}$	$\begin{cases} 43^\circ 26' 47.72'' \\ 44^\circ 8' 53.76'' \\ 47^\circ 59' 33.52'' \end{cases}$	$\begin{cases} 24^\circ 54' 55.35'' \\ 20^\circ 3' 2.70'' \\ 17^\circ 41' 48.38'' \end{cases}$	$\begin{cases} 21^\circ 57' 23.69'' \\ 21^\circ 9' 43.12'' \\ 18^\circ 58' 43.72'' \end{cases}$	$\begin{cases} 28^\circ 9' 17.86'' \\ 27^\circ 34' 28.56'' \\ 37^\circ 55' 21.53'' \end{cases}$	$\begin{cases} 20^\circ 17' 2.61'' \\ 28^\circ 39' 9.86'' \\ 24^\circ 35' 13.44'' \end{cases}$	$\begin{cases} 43^\circ 20' 52.46'' \\ 43^\circ 58' 45.21'' \\ 48^\circ 24' 1.63'' \end{cases}$	5.278617	$A + X = B + Y = C + Z$ $\sigma_1 - S_1 = 12^\circ 55.70''$
232.	$\begin{cases} a=3.6683, \\ b=3.4069, \\ c=3.5485, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2652 \end{cases}$	$\begin{cases} 43^\circ 29' 8.44'' \\ 44^\circ 11' 8.50'' \\ 48^\circ 3' 11.86'' \end{cases}$	$\begin{cases} 24^\circ 53' 44.94'' \\ 20^\circ 1' 20.91'' \\ 17^\circ 39' 27.76'' \end{cases}$	$\begin{cases} 21^\circ 55' 27.23'' \\ 21^\circ 8' 20.37'' \\ 18^\circ 56' 12.18'' \end{cases}$	$\begin{cases} 28^\circ 9' 23.74'' \\ 27^\circ 34' 40.23'' \\ 37^\circ 54' 59.69'' \end{cases}$	$\begin{cases} 20^\circ 15' 20.79'' \\ 28^\circ 37' 40.46'' \\ 24^\circ 32' 42.89'' \end{cases}$	$\begin{cases} 43^\circ 23' 20.90'' \\ 44^\circ 1' 3.59'' \\ 48^\circ 6' 7.80'' \end{cases}$	5.267735	$\sigma_1 - S_1 = 12^\circ 56.51''$
233.	$\begin{cases} a=3.6683, \\ b=3.4069, \\ c=3.5481, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2656 \end{cases}$	$\begin{cases} 43^\circ 29' 19.73'' \\ 44^\circ 11' 19.39'' \\ 48^\circ 3' 29.36'' \end{cases}$	$\begin{cases} 24^\circ 53' 39.30'' \\ 20^\circ 1' 12.74'' \\ 17^\circ 39' 16.50'' \end{cases}$	$\begin{cases} 21^\circ 55' 17.90'' \\ 21^\circ 8' 13.74'' \\ 18^\circ 56' 0.04'' \end{cases}$	$\begin{cases} 28^\circ 9' 24.20'' \\ 27^\circ 34' 40.23'' \\ 37^\circ 58' 6.96'' \end{cases}$	$\begin{cases} 20^\circ 15' 22.31'' \\ 28^\circ 37' 40.46'' \\ 24^\circ 32' 30.81'' \end{cases}$	$\begin{cases} 43^\circ 23' 32.83'' \\ 44^\circ 1' 14.68'' \\ 48^\circ 6' 24.33'' \end{cases}$	5.266861	$\sigma_1 - S_1 = 12^\circ 56.55''$ being the max. value when a, b, x, y , and $c + z$ have the values here assigned them.
234.	$\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.0007, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.8130 \end{cases}$	$\begin{cases} 48^\circ 18' 58.69'' \\ 48^\circ 50' 17.22'' \\ 55^\circ 13' 6.02'' \end{cases}$	$\begin{cases} 22^\circ 39' 30.65'' \\ 16^\circ 39' 21.14'' \\ 13^\circ 22' 11.05'' \end{cases}$	$\begin{cases} 18^\circ 7' 32.84'' \\ 24^\circ 35' 58.38'' \\ 14^\circ 19' 23.34'' \end{cases}$	$\begin{cases} 27^\circ 21' 37.97'' \\ 27^\circ 42' 7.46'' \\ 42^\circ 47' 30.97'' \end{cases}$	$\begin{cases} 25^\circ 6' 46.32'' \\ 24^\circ 45' 23.04'' \\ 19^\circ 16' 10.03'' \end{cases}$	$\begin{cases} 50^\circ 1' 39.56'' \\ 50^\circ 23' 2.84'' \\ 55^\circ 52' 9.85'' \end{cases}$	3.810566	$S_1 - \sigma_1 = 3^\circ 54' 30.32''$
235.	$\begin{cases} a=3.3457, \\ b=2.94292, \\ c=3.4569, \end{cases}$ $\begin{cases} x=4.4568 \\ y=4.85958 \\ z=4.3456 \end{cases}$	$\begin{cases} 43^\circ 56' 48.39'' \\ 45^\circ 35' 3.00'' \\ 43^\circ 35' 16.57'' \end{cases}$	$\begin{cases} 28^\circ 47' 43.41'' \\ 21^\circ 16' 52.73'' \\ 22^\circ 40' 0.20'' \end{cases}$	$\begin{cases} 25^\circ 7' 50.70'' \\ 33^\circ 30' 9.13'' \\ 25^\circ 24' 28.98'' \end{cases}$	$\begin{cases} 21^\circ 44' 56.80'' \\ 20^\circ 38' 42.63'' \\ 27^\circ 42' 7.46'' \end{cases}$	$\begin{cases} 28^\circ 18' 54.85'' \\ 26^\circ 35' 39.85'' \\ 28^\circ 39' 59.43'' \end{cases}$	$\begin{cases} 43^\circ 54' 59.18'' \\ 45^\circ 38' 14.18'' \\ 43^\circ 33' 54.60'' \end{cases}$	5.660726	$S_1 = \sigma_1$
236.	$\begin{cases} a=3.72374, \\ b=5.19556, \\ c=4.33934, \end{cases}$ $\begin{cases} x=3.672916 \\ y=4.87204 \\ z=3.44460 \end{cases}$	$\begin{cases} 22^\circ 6' 11.54'' \\ 37^\circ 0' 26.71'' \\ 20^\circ 44' 0.26'' \end{cases}$	$\begin{cases} 24^\circ 23' 47.02'' \\ 46^\circ 24' 36.87'' \\ 29^\circ 20' 57.60'' \end{cases}$	$\begin{cases} 22^\circ 51' 22.87'' \\ 43^\circ 9' 57.53'' \\ 27^\circ 38' 42.41'' \end{cases}$	$\begin{cases} 23^\circ 38' 35.69'' \\ 40^\circ 15' 6.03'' \\ 22^\circ 26' 15.45'' \end{cases}$	$\begin{cases} 18^\circ 6' 36.76'' \\ 63^\circ 45' 28.77'' \\ 21^\circ 21' 55.49'' \end{cases}$	$\begin{cases} 18^\circ 21' 7.27'' \\ 62^\circ 16' 7.63'' \\ 17^\circ 17' 6.82'' \end{cases}$	5.310628	
237.	$\begin{cases} a=3.72374, \\ b=5.19556, \\ c=4.87204, \end{cases}$ $\begin{cases} x=3.55213 \\ y=5.43034 \\ z=3.67424 \end{cases}$	$\begin{cases} 20^\circ 34' 54.15'' \\ 29^\circ 20' 57.60'' \\ 22^\circ 30' 2.36'' \end{cases}$	$\begin{cases} 25^\circ 57' 2.89'' \\ 44^\circ 36' 36.29'' \\ 37^\circ 0' 26.71'' \end{cases}$	$\begin{cases} 22^\circ 53' 21.35'' \\ 36^\circ 48' 16.75'' \\ 32^\circ 15' 48.71'' \end{cases}$	$\begin{cases} 23^\circ 38' 35.69'' \\ 37^\circ 9' 17.14'' \\ 27^\circ 14' 40.36'' \end{cases}$	$\begin{cases} 24^\circ 9' 13.62'' \\ 50^\circ 57' 43.20'' \\ 35^\circ 4' 58.08'' \end{cases}$	$\begin{cases} 25^\circ 10' 43.95'' \\ 45^\circ 47' 13.01'' \\ 27^\circ 54' 34.32'' \end{cases}$	7.30914	
238.	$\begin{cases} a=7.3782, \\ b=9.2160, \\ c=8.1664, \end{cases}$ $\begin{cases} x=6.5408 \\ y=5.4688 \\ z=5.7472 \end{cases}$	$\begin{cases} 21^\circ 50' 40.74'' \\ 20^\circ 24' 3.68'' \\ 19^\circ 16' 47.88'' \end{cases}$	$\begin{cases} 35^\circ 38' 56.37'' \\ 44^\circ 8' 38.64'' \\ 38^\circ 40' 52.69'' \end{cases}$	$\begin{cases} 26^\circ 34' 33.48'' \\ 26^\circ 11' 4.04'' \\ 29^\circ 47' 40.98'' \end{cases}$	$\begin{cases} 30^\circ 55' 3.63'' \\ 38^\circ 21' 38.28'' \\ 28^\circ 9' 59.59'' \end{cases}$	$\begin{cases} 32^\circ 24' 30.24'' \\ 46^\circ 14' 55.80'' \\ 38^\circ 37' 57.94'' \end{cases}$	$\begin{cases} 35^\circ 36' 23.16'' \\ 31^\circ 47' 33.97'' \\ 29^\circ 20' 33.45'' \end{cases}$	35.68824	See next page.
239.	$\begin{cases} a=7.5543, \\ b=8.0000, \\ c=4.7722, \end{cases}$ $\begin{cases} x=7.73156 \\ y=6.9282 \\ z=6.0000 \end{cases}$	$\begin{cases} 35^\circ 6' 22.85'' \\ 32^\circ 15' 35.19'' \\ 23^\circ 38' 1.11'' \end{cases}$	$\begin{cases} 36^\circ 31' 49.43'' \\ 35^\circ 38' 55.72'' \\ 17^\circ 49' 15.70'' \end{cases}$	$\begin{cases} 31^\circ 43' 3.17'' \\ 29^\circ 30' 9.76'' \\ 16^\circ 29' 16.00'' \end{cases}$	$\begin{cases} 39^\circ 55' 9.11'' \\ 38^\circ 24' 21.15'' \\ 28^\circ 9' 59.59'' \end{cases}$	$\begin{cases} 44^\circ 3' 56.80'' \\ 40^\circ 40' 23.11'' \\ 18^\circ 43' 47.09'' \end{cases}$	$\begin{cases} 43^\circ 24' 7.40'' \\ 36^\circ 39' 22.74'' \\ 24^\circ 27' 37.20'' \end{cases}$	28.84288	
240.	$\begin{cases} a=4.2893, \\ b=5.6794, \\ c=5.6318, \end{cases}$ $\begin{cases} x=3.931876 \\ y=3.9794 \\ z=4.1941 \end{cases}$	$\begin{cases} 20^\circ 20' 23.19'' \\ 22^\circ 24' 5.52'' \\ 23^\circ 37' 55.51'' \end{cases}$	$\begin{cases} 28^\circ 43' 4.96'' \\ 42^\circ 1' 20.41'' \\ 42^\circ 53' 10.41'' \end{cases}$	$\begin{cases} 24^\circ 20' 44.08'' \\ 29^\circ 16' 6.80'' \\ 34^\circ 30' 17.68'' \end{cases}$	$\begin{cases} 24^\circ 42' 44.07'' \\ 35^\circ 9' 19.13'' \\ 32^\circ 0' 48.24'' \end{cases}$	$\begin{cases} 26^\circ 46' 28.99'' \\ 42^\circ 48' 50.12'' \\ 44^\circ 23' 20.75'' \end{cases}$	$\begin{cases} 28^\circ 49' 49.97'' \\ 32^\circ 59' 22.04'' \\ 36^\circ 5' 31.88'' \end{cases}$	9.999143	

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